February 19, May 19, and August 19? State any assumptions you make in answering this. Compute the firm's expected total interest expense for the $20 million loan.

b. What trades should the firm make to hedge by means of a strip hedge? What trades should the firm make in order to hedge by using a rolling hedge? Suppose that ultimately, the following spot LIBOR's and futures prices are observed:

	Spot	Futures Prices		
Date	**LIBOR**	**Mar.**	**June**	**Sept.**
Feb. 19	6.00%	94.02	94.00	93.82
May 19	7.06%		92.90	92.62
Aug. 19	7.94%			91.92

What interest expense would the firm have actually paid if it did not hedge? What interest expense would it have paid if it used a strip hedge? What interest expense would it have paid if it used a rolling hedge and did all its trades on the three dates as tabulated?

10.10 Today is November 26, 2000. A corporate treasurer has noted that his firm has borrowed $25 million (face value) from a bank, and the loan is due to be repaid on December 17, 2000. The bank has told the company that it is willing to extend the loan, to March 17, 2001, at a rate equal to whatever LIBOR is on December 17, plus 100 basis points. Currently, futures prices are as follows:

Delivery Date	Futures Price
December 17, 2000	91.98
March 18, 2001	92.47
June 17, 2001	92.56
September 16, 2001	92.53

a. Should the firm hedge by buying or selling futures? Which delivery month (or months) should the firm use? How many futures contracts should it trade?

b. Suppose that on December 17, 2000, the actual three month spot LIBOR is 7.81%. If the firm did not hedge, what would be its interest expense on the new bank loan? If the firm did hedge, what would be its interest expense, after considering profits/losses on the futures contracts? What is the firm's annualized interest rate on the hedged borrowing transaction? How does it compare to the November 26 futures price?

c. Suppose instead that the firm desires to extend the loan for a year, until December 17, 2001. The new loan will be a fixed-rate, one-year loan. How many futures contracts should the firm trade on November 26 in order to hedge? Will all the futures contracts be for the same delivery month? Why or why not?

d. Suppose the firm wants to extend the loan for a year. The new loan will be a variable-rate loan, with interest rates reset quarterly. How many futures contracts should the firm trade on November 26? What factors dictate the delivery month, or months, that the firm should use?

10.11 Suppose a hedger places a stack hedge as in Table 10.5 (i.e., in terms of the number of contracts traded). However, suppose that when the hedger places the stack hedge the following futures prices are obtained:

Date	Futures Contract	Futures Price
7/28/1999	September 1999	94.555
9/13/1999	December 1999	94.320

12/13/1999	March 2000	94.220
3/13/2000	June 2000	94.170

a. Suppose the futures prices at the time of the hedge rollover are the same as they are in Table 10.5. Calculate the futures contract gain or loss.

b. Calculate the firm's net interest expense for each quarter.

c. Calculate the firm's effective loan rate for each quarter.

d. Calculate the average effective loan rate. Was this hedge more or less effective than the strip hedge presented in Table 10.4? Explain.

e. What do you notice about the futures premium (or discount) to the spot LIBOR at the time the hedge is placed?

10.12 Suppose a hedger places a stack hedge as in Table 10.5 (i.e., in terms of the number of contracts traded). However, suppose that when the hedger places the stack hedge the following futures prices are obtained:

Date	Futures Contract	Futures Price
7/28/1999	September 1999	94.555
9/13/1999	December 1999	94.320
12/13/1999	March 2000	94.220
3/13/2000	June 2000	94.170

a. Suppose the futures prices at the time of the hedge rollover are the same as they are in Table 10.5. Calculate the futures contract gain or loss.

b. Calculate the firm's net interest expense for each quarter.

c. Calculate the firm's effective loan rate for each quarter.

d. Calculate the average effective loan rate. Was this hedge more or less effective than the strip hedge presented in Table 10.4? Explain.

e. What do you notice about the futures premium (or discount) to the spot LIBOR at the time the hedge is placed?

PART 3

SWAPS

CHAPTER 11

An Introduction to Swaps

Swaps are contractual agreements between two parties to exchange cash flows. The cash flows that are swapped may be determined on the basis of interest rates, exchange rates, or the prices of indexes (such as stock indexes) or commodities. To determine the dollar amounts that will be exchanged these prices are applied to a base amount, called the **notional principal** of the swap. The two parties that agree to exchange the cash flows are called the **counterparties** of the swap. One of the counterparties will typically be a swap dealer (market maker); it is rare for two firms to negotiate the terms of a swap contract by themselves.

In this chapter, the basic features of different types of swaps are described. The **plain vanilla**, fixed-for-floating interest rate swap is covered first, in Section 11.1. A plain vanilla swap is a standard swap with no unusual features. Other interest rate swap structures are briefly covered. Currency swaps are discussed in Section 11.2, and commodity swaps in Section 11.3. Finally, the issue of credit risk in swaps is considered.

11.1 INTEREST RATE SWAPS

The fixed-for-floating fixed–floating interest rate swap is the most basic form of a swap, in which one of the parties agrees to pay (to the other counterparty) a fixed amount of money on specific dates. The fixed payments are expressed as a percentage of the swap's notional principal. For example, 8% of $20 million would create an annual payment of $1.6 million; if semiannual payments are required, the fixed amount would be $800,000 every six months. The notional principal of the swap is not exchanged; it serves only as the basis for calculating the swap payments. The percentage is the fixed interest rate, and it is multiplied by the notional principal to compute the fixed payment. The same party, who is called the fixed-rate payer, also receives a floating, or variable, amount of money on each of the specified dates. The amount to be received is also computed by multiplying a randomly fluctuating interest rate by the swap's notional principal. This floating payment is determined by a floating interest rate that changes over time, such as three-month LIBOR. If this interest rate rises, the fixed-rate payer will receive a greater floating amount of money. If the floating interest rate declines, the fixed-rate payer will receive a smaller floating amount. In practice, the amount received at each date is offset, or netted out, against the amount paid. Thus, if on one particular date, the fixed-rate payer is supposed to pay $1,600,000, and also receive $1,200,000, the net payment made is $400,000 ($1,600,000 − $1,200,000 = $400,000 outflow). When the counterparties' obligations are netted in this manner, the payment ($400,000) is called a **difference check**.

An important characteristic of most interest rate swaps is that the floating interest rate (e.g., LIBOR) that determines the floating-rate payment is set *one period before the payment date*. This means that the net payment to be made on any date is actually known one period earlier.[1] It follows that the first net payment, which will be made one period after the swap's origination date, is known on the origination date.

11.1.1 An Example of a Plain Vanilla Fixed-for-Floating Interest Rate Swap

Party A pays the fixed interest rate (and receives the floating rate). Counterparty B receives the fixed payment (and pays the floating rate). The notional principal is $40 million. The fixed rate is 7%. The fixed rate day count method is **the 30/360 day basis.**[2] The floating rate is six-month LIBOR, which is also determined on a 30/360 day basis. The swap's **origination date** is July 20, 1999, and the **termination date** is July 20, 2002. The first payment date is January 20, 2000. Semiannual payments will then be made on each July 20 and January 20, up to and including the termination date.

Assume the actual six-month LIBOR that exists on the origination date, and the six-month LIBOR that subsequently exists on all future relevant dates is as follows (of course, on the origination date only the first floating LIBOR is known).

Date	**Six-Month LIBOR**
July 20, 1999 (origination date)	6.5%
January 20, 2000	7.0%
July 20, 2000	7.3%
January 20, 2001	7.7%
July 20, 2001	7.0%
January 20, 2002	6.2%
July 20, 2002 (termination date)	5.9%

On each payment date, party A, the fixed-rate payer, must pay half of the fixed rate (half of 7%) times the notional principal of $40 million:

$$(0.07/2)(\$40 \text{ million}) = \$1.4 \text{ million}$$

Half of 7%, or 3.5%, is used to compute the fixed-rate payment because payment dates occur every six months, or half a year. If fixed payments were made annually, the fixed amount would be 7% of $40 million, or $2.8 million. If fixed payments were made quarterly, the fixed payment amount would be (0.07/4)($40 million), or $700,000. Thus, a 7% interest rate becomes a 3.5% semiannual rate, or a 1.75% quarterly rate, as appropriate.

In a typical swap, LIBOR at time $t-1$ will determine the floating payment six months later, at time t. Six-month LIBOR on July 20, 1999 determines the first floating-rate payment made on January 20, 2000 by party B to counterparty A. Thus, the floating-rate payment amount on January 20, 2000 is

$$(0.065/2)(\$40 \text{ million}) = \$1,300,000$$

TABLE 11.1 Cash Flows Resulting from the Swap[1]

Date	Six-Month LIBOR	Fixed Payment	Floating Payment	Net Cash Flow
July 20, 1999	6.5%			
January 20, 2000	7.0%	$1,400,000	$1,300,000	−$100,000
July 20, 2000	7.3%	$1,400,000	$1,400,000	$0
January 20, 2001	7.7%	$1,400,000	$1,460,000	+$ 60,000
July 20, 2001	7.0%	$1,400,000	$1,540,000	+$140,000
January 20, 2002	6.2%	$1,400,000	$1,400,000	$0
July 20, 2002	5.9%	$1,400,000	$1,240,000	−$160,000

[1]Note how the floating payment at time t is determined by the interest rate at time $t-1$. The net cash flows are the difference between the fixed and floating payments. A negative net cash flow means that the fixed-rate payer is making a cash payment to the receive–fixed counterparty.

The cash flows that are exchanged are presented in Table 11.1. Party A is the fixed-rate payer, who pays $1,400,000 on each of the payment dates and receives the amounts in the "Floating Payment" column on each payment date. The cash flows are netted, so that only the amounts in the column labeled "Net Cash Flow" are exchanged. The amounts in the last column are denoted as "–" or "+" from the fixed-rate payer's viewpoint. Thus, party A will pay (a "–" sign) $100,000 on January 20, 2000, and receive (a "+" sign) $60,000 on January 20, 2001.

Note again that the LIBOR that exists at time $t-1$ determines the floating cash flow at time t. Therefore, net cash flows are always known six months in advance. On July 20, 2000, six-month LIBOR is 7.3%. This locks in the floating payment that will be made six months later, on January 20, 2001. The floating payment on January 20, 2001 will be (0.073/2)($40,000,000) = $1,460,000.[3]

The cash flows arising from the swap may be depicted through the use of a cash flow diagram as shown in Figure 11.1. The arrows pointing up represent the cash inflows for the fixed-rate payer; they are the floating payments. The arrows pointing down represent the fixed payments of $1,400,000 made every six months by the fixed-rate payer (party A).

11.1.2 Generalizing the Plain Vanilla Fixed–Floating Interest Rate Swap

We can generalize the structure of a plain vanilla fixed-for-floating interest rate swap as follows. Define:

NP = notional principal

h_f = unannualized fixed rate

$\tilde{h}_t$ = variable (floating) unannualized rate, such as LIBOR, that exists at time t

The term "unannualized" refers to the practice of converting an annual interest rate to a partial-year rate. If there are t days between payments, and if there are 365 days in a year then

$h=(r)(t/365)$. The tilde (~) that is placed above the floating interest rate means that on the origination date, the floating rate is unknown (it is a random variable characterized by a probability distribution) on all dates beyond time 0. Then, the fixed-rate payer must pay $(h_f)(NP)$ at each payment date and will receive $(\tilde{h}_t)(NP)$ at payment date $t+1$. Figure 11.2 illustrates the cash flow structure for a three-year standard fixed–floating interest rate swap with semi-annual payments.

Note in Figure 11.2 that the first "floating" payment is not a random variable. The time 0 floating interest rate determines the time 1 floating cash flow. The floating payments occurring beyond time 1 are unknown as of the swap's origination date. Each net payment is always known one period before the exchange of cash flows.

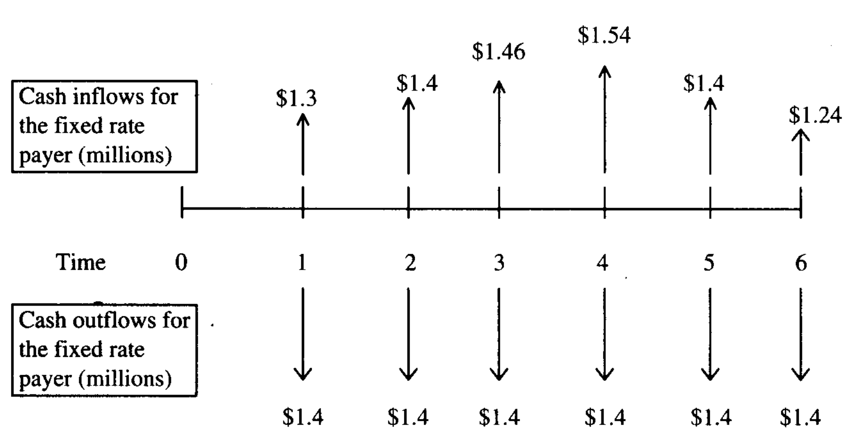

Figure 11.1 Cash flow diagram for the swap example. The cash flows are netted against each other to produce one payment from the losing party to the winning party.

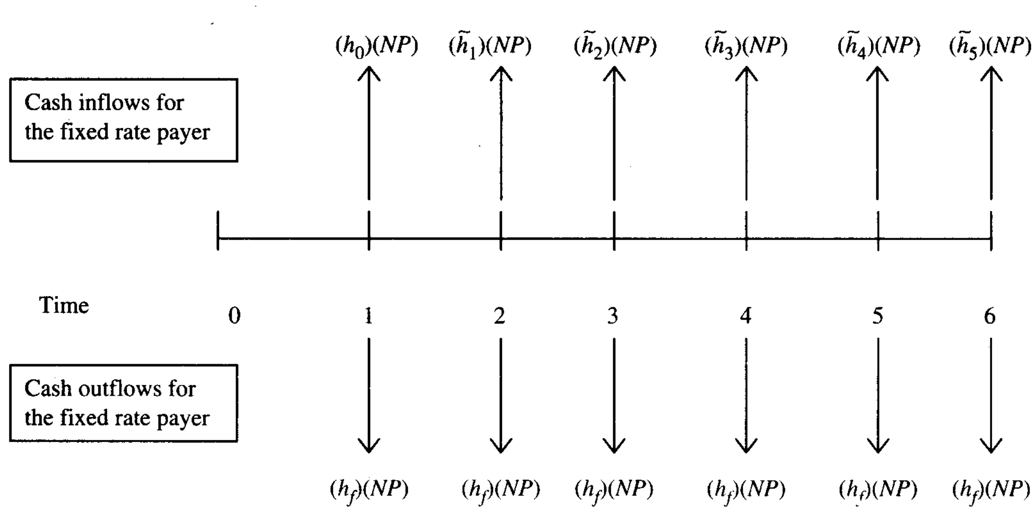

Figure 11.2 Cash flow diagram for a plain vanilla interest rate swap.

11.1.3 Characterizing a Fixed–Floating Interest Rate Swap as a Portfolio of FRAs

A fixed–floating interest rate swap contract may be more easily understood by realizing that it is virtually equivalent to a portfolio of forward contracts, each of which has a different settlement (termination) date, and each of which has the same forward rate. Thus, instead of entering into a fixed–floating swap as the receive-fixed party (this is the party that pays the floating rate and receives the fixed rate), a firm or institution could instead sell a portfolio of forward rate agreements. In a swap, if the floating interest rate is below the fixed rate, the receive-fixed party will benefit, because the floating cash flow amounts paid are less than the fixed rate payments received; the difference check will be paid by the pay-fixed party to the receive-fixed counterparty at each settlement date. Similarly, at each settlement date of a FRA, if the spot interest rate is below the contract rate (interest rates have declined), the seller of the FRA will receive a payment from the party that bought the FRA.

Table 11.2 summarizes the analogy between a fixed–floating interest rate swap and a portfolio of forwards.

For example, consider the swap described in Section 11.1.1. The last column of Table 11.1 shows the cash flows for the pay-fixed party, so the cash flows for the receive-fixed counterparty will just be the negative of those shown. Instead, the receive-fixed counterparty could have sold a strip of five FRAs: a 6×12 FRA, a 12×18 FRA, an 18×24 FRA, a 24×30 FRA, and a 30×36 FRA. Recall that an $N\times M$ FRA covers an interest rate for the period beginning at month N and ending at month M. The contract period is $(M-N)$ months. The first cash flow of the swap, on January 20, 2000, does not require a FRA because that payment is predetermined on the swap's origination date. The receive-fixed party knows that \$100,000 will be received on January 20, 2000.

Recall that Equation (3.1) is used to determine the settlement cash flow of a standard FRA:

$$\pi = \left|\frac{P[r(t1,t2) - fr(0,t1,t2)](D/B)}{1+[r(t1,t2)(D/B)]}\right| \tag{3.1}$$

settlement amount = [$P\times$(difference between settlement rate and contract forward rate)
$\times$ contract period]/[1 + (settlement rate $\times$ contract period)]

The contract period in this formula is a fraction of a year. It converts the annualized rates [$r(t1, t2)$ and $fr(0, t1, t2)$] into unannualized rates.

To establish equivalency between the strip of FRAs and the swap, each of the five FRAs should have the same contract rate of 7%, and the length of each contract period is six months (which is 0.5 year when the 30/360 day count method is used). The January 20, 2000, settlement cash flow comes from the 6×12 FRA with a notional principal of \$40 million. On January 20, 2000,

TABLE 11.2 An Interest Rate Swap Is Analogous to a Strip of FRAs

Swap Party	FRA Party	Receives Settlement Payments
Pay-fixed	Buy a portfolio of FRAs	If interest rates rise
Receive-fixed	Sell a portfolio of FRAs	If interest rates decline

six-month LIBOR is 7.0%. The settlement amount is therefore \$0 (both the settlement rate and the contract forward rate are 7.0%, so the difference between them is 0.00). Note that this is the same net cash flow for the swap *six months later, on July 20, 2000.*

On July 20, 2000, six-month LIBOR is 7.3%. Therefore, the settlement amount paid by the seller of the FRA is:

$$\frac{(\$40,000,000)(0.073-0.07)0.5}{1+(0.073)(0.5)}=\$57,887.12$$

This settlement amount, \$57,887.12, is the present value of \$60,000.[4] Note that undiscounted amount of \$60,000 is the cash flow paid by the receive-fixed counterparty of the *swap six months later, on January 20, 2001.*

Table 11.3 summarizes the cash flows arising from the swap (from the receive-fixed party's view), and from a party who sells a series of five FRAs, each with the same contract rate of 7%.

The lines are drawn in Table 11.3 connecting the swap net cash flows to the corresponding FRA settlement payments. The differences between the cash flows from the two contracts arise from their different standard settlement features. A typical FRA uses the spot floating rate on the settlement date to determine the cash flow, which is discounted. A typical swap uses the time $t-1$ spot floating rate to determine the time t cash flow, and it is not discounted. However, recall that the parties to over-the-counter derivatives contracts can negotiate terms. An arrears reset swap (see note 1 in this chapter) is more similar to a portfolio of FRAs because it uses the time t spot floating rate (rather than the time $t-1$ spot rate) to determine the floating-rate payment. FRAs that do not have the cash flows discounted can also be negotiated, although these would be defined as being "nonstandard" contracts. The other important difference between a strip of FRAs and a swap is that each of the FRAs will have a unique contract rate, while the fixed rate of a swap is applied to each of its cash flows. Basically, the swap's fixed rate is an "average" of the different contract rates of the strip of FRAs. We will cover swap pricing in Chapter 13.

What is important to know is that the cash flows arising from a swap and a portfolio of forwards differ only because of their different settlement conventions. Conceptually, they are equivalent.

TABLE 11.3 Summary of Cash Flows Arising from the Swap and from the Sale of Five FRAs[1]

Date	Six-Month LIBOR	Swap Net Cash Flow	FRA Settlement Payment	FRA
July 20, 1999	6.5%			
January 20, 2000	7.0%	+\$100,000	\$0	6×12
July 20, 2000	7.3%	\$0	−\$ 57,887.12	12×18
January 20, 2001	7.7%	−\$ 60,000	−\$134,809.82	18×24
July 20, 2001	7.0%	−\$140,000	\$0	24 X 30
January 20, 2002	6.2%	\$0	+\$155,189.14	30×36
July 20, 2002	5.9%	+\$160,000		

[1]The interest rate at time t determines a swap's cash flow paid at time $t+1$. But in a FRA, the time t interest rate determines the discounted cash flow at time t.

In general, a firm should use a FRA when it is managing a single-period interest rate risk (a FRA is essentially a single-period swap). A swap will be preferred when the interest rate risk exists at several periodic (quarterly, semiannually, etc.) future points in time.

There are other elements that make a swap different from a portfolio of FRAs, and these should be considered when one is deciding which contract to use. The credit risk arising from dealing with a counterparty is relevant. The liquidity of the two contracts must be analyzed. It is also important to compare the quoted prices to the contracts' theoretical prices, so that the cheaper contracts are bought (or the higher priced contracts are sold). In theory, after the settlement differences, default risk, and liquidity have been accounted for, the values of the swap and the portfolio of FRAs should be identical, but this does not mean that the actual prices (by different dealers, in particular) will be the same. Relative supply and demand in different markets in the short-run can create price differentials. The principles of pricing swaps are covered in Chapter 13.

Finally, firms should carefully examine the accounting treatment of swaps vs FRAs, any regulatory considerations that might permit one but not the other (e.g., some firms and institutions are prohibited from using futures and options, but not swaps), and whether the tax treatments of the two derivatives would differ.

11.1.4 Characterizing a Fixed–Floating Interest Rate Swap as a Coupon-Bearing Asset Plus a Coupon-Bearing Liability

Engaging in a fixed-for-floating interest rate swap is tantamount to creating a new asset, funded with a new liability. The pay-fixed party to the swap has effectively issued a coupon bond (a liability). In both a swap and a coupon bond, a fixed amount must be paid periodically (almost always every six months in the case of the corporate bond). The only difference is that the firm still owes the principal when it issues a bond, while the notional principal is not owed in an interest rate swap.

The pay-fixed party to the swap has also effectively purchased a floating-rate bond (an asset), where the interest received changes every period. Again, you should remember that the pay-fixed swap party does not receive the notional principal payment at the swap's termination, whereas the owner of a floating rate bond does receive principal when the bond matures.

Another difference exists regarding the accounting treatment of the transactions. The financial assets will appear on a firm's balance sheet. As of late 1996, swaps were considered to be off–balance-sheet financial instruments; thus, you would not see swaps on either the asset side or the liability side of a firm's balance sheet.

11.1.5 Quoting Prices of Plain Vanilla, Fixed–Floating Interest Rate Swaps

As the swap market developed, many conventions evolved that facilitated the quoting of prices. For each floating rate (three-month LIBOR, six-month LIBOR, etc.), the price of a fixed–floating interest rate swap is quoted for different maturities. Standard maturities are 2, 3, 5, 7, and 10 years. Another name for the maturity of a swap is its **tenor**. Thus, for three-month LIBOR, a set of prices will exist, depending on whether the swap tenor will be two years, three years, etc. A different set of prices will be quoted for two-year, three-year, etc. swaps, when the floating rate is six-month LIBOR. When a firm desires a swap tenor that is not quoted, its spread is interpolated from the two maturities that are quoted and then added to the yield of a Treasury instrument with a maturity equal to the desired tenor.

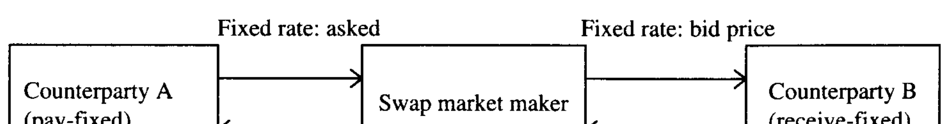

Figure 11.3 The swap dealer receives the fixed asked price and pays the fixed bid price.

Market makers quote prices in terms of the fixed rate. Essentially, the floating rate is either bought or sold, and the price paid is the fixed rate. When prices change, the change will appear as a change in the quoted fixed rate.

Most typically, swap rates are quoted as a spread, or margin, over the Treasury note rate with the same time to maturity as the swap's tenor.[5] Thus, if the yield to maturity for a five-year Treasury note is 6.60%, a swap market maker might quote the swap spread as 27/24 over 6.60%. This means that the pay-fixed party must pay a fixed interest rate of 6.87% (27 basis points above 6.60%) and will receive LIBOR. A firm that wishes to be the receive-fixed counterparty will receive a fixed interest rate of 6.84% (24 basis points above 6.60%) and pay LIBOR. The swap market maker earns the bid–asked spread of three basis points as a profit. Under typical market conventions, the 27-basis-point swap spread quote is the offer price, or asked price. The quote of 24 basis points is the bid swap price. Figure 11.3 illustrates the nature of these quotes.

Dealers will quote wider spread quotes (e.g., 30/20) for less creditworthy counterparties. The wider spreads serve as risk premiums that compensate the dealer for bearing credit (default) risk. Thus, the dealer will both pay a lower fixed rate (when acting as the pay-fixed party), and receive a higher fixed rate (when acting as the receive-fixed party) when dealing with riskier counterparties. When swap dealers are dealing with counterparties they feel are more likely to default, they may also require collateral (equivalent to initial margin for a futures contract), ask for a letter of credit, require the counterparty to pay for swap insurance, or demand that the swap be marked to market.

The floating rate is said to be quoted **flat**, with no margin above any floating rate index.

About 75% of all U.S. dollar based interest rate swaps are based on LIBOR. Other U.S. floating rates that are swapped and quoted, include commercial paper rates, the Fed funds rate, the prime rate, and Treasury bill rates. Still other fixed–floating interest rate swaps are based on foreign interest rates such as sterling LIBOR (for British pounds), yen LIBOR, and Euribor (the Euro Interbank Offered Rate) for euros.[6] In plain vanilla fixed–floating interest rate swaps, each of these floating rates is swapped against the respective fixed rates in the same currency. Many other swap structures exist, including swaps in which a fixed rate in one currency is exchanged for a floating rate denominated in a different currency.

11.1.6 Other Interest Rate Swap Structures

Many other interest rate swaps exist, other than plain vanilla fixed–floating interest rate swaps. In this section, just a few of the more common ones are discussed.

Ignoring the bid–asked spread, fairly priced **at-market** swaps have no value when they are originated.[7] In an **off-market swap** (also called an **off-market coupon swap**), the fixed rate is "away" from the market. For example, a swap dealer may be quoting a price of 30 basis points over the seven-year Treasury rate of 7.04% for a swap with a tenor of seven years. However, a firm might not want to pay 7.34% fixed; it may want to pay only 6.5%, and still receive LIBOR. In this case, an initial payment from the firm to the dealer will have to be negotiated. The payment reflects the value of only paying 6.5% fixed, when the current market price is 7.34%. Another firm might wish to pay 8% fixed. In this case, the dealer will make an initial payment to the firm.

Firms will desire off-market swaps to hedge an existing interest rate risk precisely. For example, a mutual fund that owns an 8% coupon bond and expects interest rates to rise might wish to convert this bond into a floating-rate bond. Even though current swap quotes are below 8%, it might want to pay a fixed rate of 8%.

Another firm may wish to unwind a swap entered into earlier in which it was receiving a fixed interest rate of 6.5%. It will then wish to pay a fixed rate of only 6.5% at a time when an at-market swap might require that this firm pay a fixed rate of 7.34%. By entering into this off-market swap, the firm will effectively offset its old swap. Note that the off-market swap should have a tenor equal to the remaining time to maturity of its old swap. In both swaps, the same LIBOR serves as the floating rate.

In some swaps, the notional principal will change according to a predetermined schedule; these are called **amortizing swaps**.[8] In an **index-amortizing swap**, the rate at which the notional principal declines is based on some interest rate index that changes randomly over time. For example, if LIBOR declines to some level, the notional principal may drop to 75% of its original level. Index-amortizing swaps are used in mortgage finance to deal with prepayment risk. When interest rates decline, homeowners will typically increase their prepayments of mortgages. When mortgages are paid off early, the principal underlying a mortgage-backed security will decline. Index-amortizing swaps can be used to hedge this prepayment risk faced by the owners of mortgage backed securities.

A **basis swap** is one in which both interest rates float. For example, three-month LIBOR might be exchanged for the five-year Treasury rate, or the prime rate might be exchanged for the three-month commercial paper rate. At each settlement date, both floating rates are observed, and when multiplied by the swap's notional principal, the two cash flows are determined. Instead of one rate being held constant, as the fixed rate is in a fixed–floating swap, both rates randomly change as time passes. These are also called **floating–floating swaps**. When one or both of the floating rates is a long-term rate, the basis swap may be called a **yield curve swap**. Basis swaps allow speculators to bet on a forecasted shift in the yield curve. Another use arises if a firm has a floating-rate liability; the basis swap allows it to change the index rate that determines its floating-rate payments from one rate to another.

A firm may wish to execute a swap contract today but not begin exchanging payments until a date in the distant future. This is called a **forward swap**. Alternatively, another firm may want to have the option to enter into a swap at some future date, only if interest rates have changed to its benefit; such options on swaps are called **swaptions**. A swaption may give the firm the right but not the obligation to enter into a swap as a fixed-rate payer any time in the future, up to some stated date, or it may give the firm the right to enter into a swap as a receive-fixed party. **Callable swaps** give the pay-fixed party the right to exit an existing swap contract, and **putable swaps** give the receive-fixed party the right to exit an existing swap.

11.2 Currency Swaps

In a currency swap, two different currencies are periodically exchanged. One important difference between interest rate swaps (which are in one currency) and currency swaps (which are in two different currencies) is that in the latter, the principal amounts are *typically* exchanged, both on the origination date and at maturity, based on the *initial* spot exchange rate.[9]

Exchanging the principal amounts at the swap's origination at the prevailing spot exchange rate has no valuation consequences. Suppose that the exchange rate is ¥104/$. If two parties agree to exchange ¥104 million for $1 million, the exchange is of no value for either party. Because of this, we can conclude that a currency swap is equivalent to a firm issuing a coupon bond in one currency (a liability) and buying a coupon bond (with an equal value to its liability) in the other currency.

There are actually four types of basic currency swaps:

1. Fixed (in one currency) for fixed (in the other currency)
2. Fixed for floating
3. Floating for fixed
4. Floating for floating

11.2.1 An Example of a Fixed–Fixed Currency Swap

Define P_1 and P_2 as the initial principal amounts in currencies 1 and 2, respectively (e.g., $P_1 = \$10$ million, $P_2 = ¥1040$ million). Also define r_1 and r_2 as the fixed interest rates in currencies 1 and 2, respectively (e.g., $r_1 = 5\%$, $r_2 = 1\%$). Note that r_2 is the interest rate earned on Japanese yen. The swap terminates three years hence. Payments are semiannual, on a 30/360 day count basis. The first payment is six months hence, and the last is on the termination date.

Currency swaps have three stages to their cash flow exchanges. First, at origination, party A gives $10 million to counterparty B, and receives ¥1040 million from counterparty B. In the second stage, periodic payments are swapped. In the third and last stage, party A returns ¥1040 million to B, and B gives $10 million back to party A.

Note that typically, the spot exchange rate on the swap's commencement date determines the amounts exchanged *both* initially (stage 1) and at maturity (stage 3). In this example, the spot exchange rate at origination is ¥104/$ (which equals $0.009615/¥). In the second stage, there are periodic exchanges of currencies. Party A gives to B ¥5.2 million every six months, because (0.01/2)(¥1040 million) = ¥5.2 million. Counterparty B gives $250,000 to party A, because (0.05/2)($10 million) = $250,000. Figure 11.4 illustrates the three stages to this fixed–fixed currency swap.

11.2.2 An Example of a Fixed–Floating Currency Swap

In this swap, which has a tenor of three years, $P_1 = ¥2080$ million, and r_1 is a fixed rate of 1% in yen; $P_2 = \$20$ million, and r_2 is a floating rate of six-month LIBOR, denominated in dollars. Settlement dates are every six months, beginning six months hence. On the origination date, six-month LIBOR is 5.5%. At subsequent dates, six-month LIBOR is tabulated as follows:

Time	6-Month LIBOR
0.5	5.25%
1.0	5.5%
1.5	6%
2.0	6.2%
2.5	6.44%

On the origination date, the fixed-rate payer pays $20 million to the fixed-rate receiver. The fixed-rate receiver pays ¥2080 million to the fixed-rate payer.

The stage 2 cash flows occur every six months. Table 11.4 shows the actual cash flows that are exchanged on all settlement dates of the swap. As with an interest rate swap, the floating rate at time $t-1$ determines the floating rate payment at time t. For example, because six-month LIBOR on the origination date is 5.5%, the dollar cash flow six months later is (0.055/2)($20 million) = $550,000. On the termination date (time 3.0 in Table 11.4), the time 2.5 LIBOR of 6.44% determines periodic payment of $644,000 [(0.0644/2)($20,000,000) = $644,000]. In addition, the principal amounts are again swapped at maturity. The pay-fixed party gives ¥2080 million to counterparty B; B gives $20 million back to party A.

Figure 11.5 is a cash flow diagram that illustrates the cash inflows and outflows from the viewpoint of party A (the pay-fixed party, who is paying fixed in yen, and receiving floating dollar periodic payments).

Note again that the time $t-1$ floating rate is used to determine the floating cash flow at time t. Also note that the termination date cash flows include the original principal amounts. These are

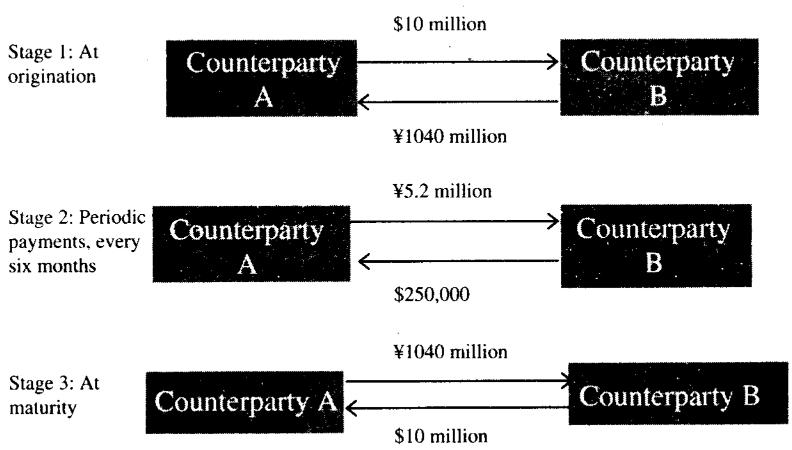

Figure 11.4 The three stages of a fixed–fixed currency swap.

TABLE 11.4 Payments That Arise as Part of a Fixed–Floating Currency Swap

Time	6-Month LIBOR	Fixed Rate Payment	Floating Rate Payment
0	5.5%	$20 million	¥2080 million
0.5	5.25%	¥10.4 million	$550,000
1.0	5.5%	¥10.4 million	$525,000
1.5	6%	¥10.4 million	$550,000
2.0	6.2%	¥10.4 million	$600,000
2.5	6.44%	¥10.4 million	$620,000
3.0		¥10.4 million	$644,000
		+¥2080 million	+$20 million

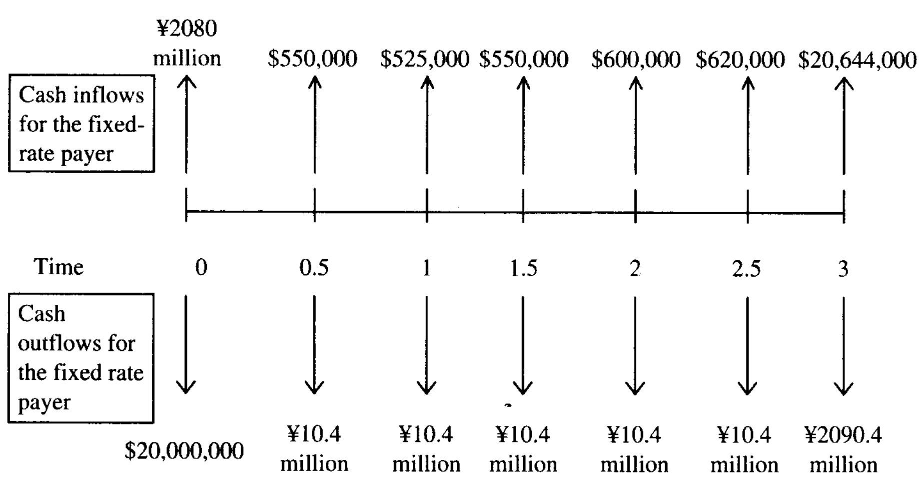

Figure 11.5 Cash flows arising from a fixed-for-floating currency swap.

typical of plain vanilla fixed–floating currency swaps, but other structures can be negotiated, including those in which only the initial principal amounts are swapped (but not at maturity), or no principal amounts are swapped. And, as with in-arrears interest rate swaps, a firm can request that the time t floating interest rate establish the floating cash flow at time t.

11.2.3 Other Currency Swap Structures

In a standard currency swap, principal amounts are exchanged at the swap's origination and at maturity. When principal amounts are not exchanged either at origination or at maturity, a fixed-fixed currency swap is called a **coupon-only swap**, or an **annuity swap**.

In Section 11.1.6, off-market, amortizing, and forward swaps, and swaptions were discussed in relation to interest rate swaps. These variations exist for currency swaps, too. When the current floating interest rate (or interest rates) is used to establish current settlement amounts, the swap is called an in-arrears swap, and this form exists for both interest rates and currency swaps, as well.

Differential swaps are more commonly called **diff swaps**. A diff swap is a floating–floating currency swap, except both floating rates are applied to the notional principal of *just one of the currencies*, and *the swapped payments are in that single currency.* Diff swaps allow a firm that is engaging in a currency swap to eliminate currency risk exposure. Because only one currency is involved, the notional principal is not exchanged, and the exchanged cash flows are netted, so only one difference check is paid from one party to the other.

For example, a firm may wish to make semiannual payments based on six-month yen LIBOR, receive six-month U.S. dollar LIBOR, and have both the payments and receipts denominated in dollars. Because current Japanese interest rates are below U.S. interest rates, the firm will have to pay yen LIBOR plus a spread. Assume the spread is 400 basis points.[10] At each settlement date, both yen LIBOR and U.S. dollar LIBOR are observed, and these are netted to determine the net payment or receipt, in dollars. The swap's tenor is two years, payments are semiannual, and the day count method is 30/360. Assume that rates are determined as in an in-arrears swap, so that the two time t floating interest rates determine the time t cash flows. The notional principal of the swap is $10 million.

Six months hence, six-month U.S. LIBOR is 5.6% and six-month yen LIBOR is 1.2%. The firm is required to pay [((0.012 + 0.04)/2)($10,000,000)] = $260,000. The firm will receive $280,000, or (0.056/2)($10,000,000). The difference check received is $20,000.

11.3 COMMODITY SWAPS

Commodity swaps are contracts in which the counterparties agree to exchange cash flows that are determined on the basis of commodity prices. They are equivalent to a strip of forward contracts on a commodity, each of which has a different maturity date, and the same forward price. The most typical underlying commodities are crude oil and other energy-related products. To a much lesser extent, some commodity swaps exist using precious metals such as gold and silver, other metals such as copper, and some agricultural products such as wheat. Most commodity swaps are fixed-for-floating swaps based on one commodity product.[11] One party agrees to pay a fixed price for a notional principal of the commodity, at a set of future dates. The other counterparty agrees to pay a floating price. The notional principal of the commodity is not exchanged. Commodity swaps require that the quantity and quality of the commodity, as it is priced at a specific location, be defined.

For example, consider a commodity swap involving a notional principal of 100,000 barrels of crude oil. One party agrees to make fixed semiannual payments at a fixed price of $21/bbl, and receive floating payments. Payments are in dollars (they could be in any currency).

On the first settlement date, if the spot price of crude oil is $20.46/bbl, the pay-fixed party must pay ($21/bbl)(100,000 bbl) = $2,100,000. The pay-fixed party also receives ($20.46/bbl) (100,000 bbl) = $2,046,000. The net payment made (a cash outflow for the pay-fixed party) is then $54,000.

Commodity swaps allow producers and users of different commodities to fix the prices they will receive or must pay for the commodities on each of several dates in the future.

11.4 EQUITY INDEX SWAPS

In equity index swaps, frequently called **equity swaps**, the counterparties pay and receive cash flows based on the rates of return of a stock market index (against either a fixed or floating interest rate), or two stock market indexes. In some equity swaps, the notional principal of stock may be a narrowly defined index (such as a major oil company stock index), or just an institution's portfolio. A party may wish to pay the return on its stock portfolio and receive LIBOR, and it will engage in this swap instead of actually selling the stocks in its portfolio.

It is interesting to note that the return on a stock index could be negative. If this happens, the party that receives the return on the stock index will actually receive a negative return; that is, this party will have to *pay* the negative stock index return *and* also pay the interest rate.

Equity index swaps have become popular for several reasons. They allow institutions to gain exposure to stock markets in countries where the governments have created high entry costs. They let equity money managers quickly execute asset allocation decisions at low cost. They can be used to customize exchange rate exposure, since the payments may be made in any desired currencies. Equity index swaps also allow securities dealers who finance their portfolios with borrowed funds (at LIBOR) to hedge against unexpected changes in the value of their portfolios.

Finally, equity index swaps have been used to circumvent tax laws in some cases. Suppose you own a stock portfolio that has appreciated in value. You have now turned bearish and wish to sell your portfolio, but if you do, you will have to pay a great deal in capital gains taxes. Instead, you can engage in an equity swap and agree to pay the rate of return on your portfolio and receive LIBOR. However, changes in the tax laws effective in 1997 have reportedly reduced the advantages swapping your portfolio's returns to avoid taxes.

11.5 Credit Risk in Swaps and Credit Swaps

A basic definition of credit risk is the risk that one of the parties to a transaction will default, and fail to abide by the contractual obligations in the contract. The most simple form of credit risk occurs when the issuer of a bond fails to make the coupon payments or fails to repay the bond's principal. The maximum amount exposed to a bond's credit risk is its market value (the present value of its remaining coupon payments and its principal).

When at-market swaps are first originated, the initial amount at risk should one of the parties default is zero. As will be seen in Chapter 13, swaps are priced to be zero net present value transactions for both parties, except for a (typically) small profit for the swap dealer, created by the bid–asked spread.

However, after its origination, the swap will likely become a valuable asset for one of the parties, which means that it will assume a negative value (become a liability) for the other counterparty. When the swap becomes a liability for one of the parties, the other (the party for whom it has become an asset) will face credit, or default, risk.[12] Default risk is the exposure to the possibility that the other (losing) party will default, and not make its future expected net payments. In other words, the present value of the expected payments to be made by the party likely to default exceeds the present value of its expected receipts. When the swap is originated, it is impossible to know who will subsequently face the credit risk.

Firms will default on swaps when they fall into financial distress and cannot fulfill the terms of a swap. However, swap dealers are not very concerned with a customer's financial situation if the swap is an asset for the counterparty and a liability for the swap dealer. In other words, the credit risk faced by a swap dealer depends both on the financial condition of its customer and how interest rates or foreign exchange rates have changed since the swap's origination. Changes in financial prices create positive swap value for only one party and negative swap value for the other. Generally, a swap dealer will be concerned about the other party defaulting only when the swap is an asset for itself.

Consider a plain vanilla interest rate swap with a tenor of three years. When it was originated, party A agreed to pay 8% fixed, and to receive six-month LIBOR. Suppose that six months after origination, the price of a swap with a tenor of *2½ years* is 7.2% (in exchange for six-month LIBOR). This swap has now become a liability for party A and an asset for the swap dealer. The swap dealer faces default risk because the decline in the price has created an incentive for party A to default. If it could walk away from its original swap contract and enter into a new one, A could save 80 basis points per year, for 2½ years, as applied to the swap's notional principal. If party A is facing financial difficulties, the market will recognize the greater default risk, and the value of the swap for the dealer is less.

This example illustrates one method of assessing the amount of a swap that is really at risk. Recall that prices of interest rate swaps are quoted as a fixed rate. It is the floating rate that is bought or sold in an interest rate swap. Thus, credit risk can be measured by comparing the price of the old swap to the price of a replacement swap. If the prices differ, the swap is an asset for one party and a liability for the other. The party for whom the swap now has a negative value has the incentive to default. The party for whom the swap now has positive value faces the default risk.

In comparison to a plain vanilla interest rate swap or to currency swaps in which there is no exchange of principal at maturity, there is greater potential default risk with a currency swap that requires a final exchange of principal. The increased default risk arises because the contractual obligation to exchange the (relatively) large principal amounts will likely create a greater buildup in value if exchange rates change during the life of the swap.

For example, in the fixed–floating currency swap in Section 11.2.2, the fixed-rate payer is obligated to exchange ¥2080 million and receive $20 million at the maturity of the swap. This implies that the exchange rate at the swap's origination was ¥104/$. If the yen price of a dollar declines, the fixed-rate payer will have a greater incentive to default. Under the terms of the swap, he must pay ¥104/$. If the exchange rate declines to, say, ¥85/$, the pay-fixed party has a large liability, and an incentive to default on his contractual obligations.[13]

Swap dealers will use different approaches to manage the credit risk they face. As discussed in Section 11.1.5, swap dealers will price the credit and default risk they perceive by quoting wider spreads when they quote prices to prospective customers. Some institutions are willing to assume the credit risk that exists in completed swap contracts. Some institutions will insure swap contract performance for a fee. **Credit enhancements** refer to swap terms that are designed to compensate swap dealers for the risk that a counterparty will experience credit deterioration, hence greater default risk. Swap dealers may demand collateral, a letter of credit, and/or that the swap be marked to market. Less creditworthy parties to a swap must also face the possibility that a financially strong swap dealer will falter. This happened to customers of Enron (which was a swap dealer) in 2001.

Finally, note that the terms of swap contracts do not allow a party to default only on the swaps that have become liabilities. **Netting agreements** have become standard. Netting means that if a party defaults on a swap that has become a liability, the swap dealer does not have to make payments on any swaps that have become positive value assets for that defaulting party. Netting agreements mean that swap dealers can use portfolio approaches for measuring the default risk they face with any one counterparty.

Credit derivatives represent an innovative approach to managing credit risk. Credit derivatives are derivative contracts with payoffs that are determined by some credit event. Thus, a credit derivative may pay off if a party (a firm, sovereign government, etc.) experiences a bond rating decline, if some asset experiences a price decline because of increased credit risk, or if the party misses a scheduled payment (defaults) on a financial contract. There are swaps, forwards, and options with payoffs that are determined by credit events. All are custom-made OTC contracts.

The **credit swap** is a common credit derivative. The buyer of a credit swap is the party that pays a fixed payment each period (an annuity) to the counterparty. The buyer receives nothing at each date as long as the credit event has not occurred. If the credit event occurs, the buyer receives a contractually specified dollar amount. For example, a bank doing a great deal of business with Riscorp might be concerned about the impact on itself if Riscorp's credit quality declines. This bank might buy a credit swap. In the credit swap, it agrees to pay basis points of the notional principal amount to the swap dealer. The swap dealer will make a payment to the bank only if Riscorp is downgraded by Moody's or an other rating agency (the credit event).

A **total return swap** is also a credit derivative. As an example of a total return swap, consider a mutual fund that owns some bonds of the BBB Corporation. The mutual fund is required to hold only investment grade bonds, and it wishes to hedge against any possible further credit deterioration in the BBB Corp.[14] The swap dealer agrees to pay a fixed or floating interest rate to the mutual fund. In turn, the mutual fund agrees to pay the periodic rate of return on the bonds to the dealer. The periodic rate of return consists of both capital gains/losses and the income yield:

$$\text{periodic rate of return} = \frac{P_t - P_{t-1} + C}{P_{t-1}}$$

where P_t is the security price at the end of the period, P_{t-1} is its price at the beginning of the period, and C is the interest paid during the period.

If BBB Corp. experiences a decline in its credit quality, then its bond prices will decline and the periodic rate of return will be low or negative. In this way, the mutual fund is compensated for the credit risk it faces.

Credit derivatives such as credit swaps permit intermediaries to strip out credit risk from a financial instrument. Thus, investors can customize financial products and retain the amount of credit risk they desire. An expert bond manager who wishes to manage her portfolio based on her interest rate forecasts can do so without worrying about changes in the bonds' default risk premia.

The credit swap market grew rapidly from zero in the 1990s. By the first quarter of 2001, the Office of the Comptroller of the Currency estimated that the notional principal of *credit derivatives* was $426 billion (www.occ.treas.gov/deriv/deriv.htm). *RISK,* a publication devoted to financial risk management, claimed that early in 2001, the notional value of outstanding credit derivatives transactions was close to $1 trillion.[15]

The International Swaps and Derivatives Association (ISDA) has published credit swap documentation, which can be downloaded (www.isda.org/). A website devoted to credit derivatives is www.margrabe.com/CreditDerivatives.html. ISDA is a global trade association made up of over 450 members, including 208 "primary members" (the swap dealers) and 120 "subscriber members" (the corporate, sovereign, and supranational end users of swaps); many dealers are also end users. The remaining members, called "associate members," consist of software, accounting, consulting, and legal firms that are active in the OTC derivatives market.

We will return to the topic of credit derivatives again in Chapter 20.

11.6 Summary

This chapter provides a broad overview of the nature of common swaps of several types. A swap contract is an agreement between two counterparties to exchange cash flows. The amounts exchanged depend on interest rates (interest rate swaps), foreign exchange rates (currency swaps), the prices of commodities (commodity swaps), or the level of stock indexes (equity index swaps).

A swap contract creates an asset and a liability for each party. A swap will involve the contractual promise to make payments (a liability) and to receive payments (an asset). A swap is also equivalent to a portfolio of forward contracts, each with a different maturity date.

The notional principal of interest rate swaps is not exchanged at origination or at maturity. However, the principal of currency swaps is usually exchanged both at origination and at maturity. In a currency swap, the values of the amounts swapped are as a rule equal at origination because they are based on the initial spot exchange rate. At maturity, the same amounts, based on the *initial* exchange rate, are again exchanged. Typically, the rate observed at the start of a period determines the floating rate cash flow that will be exchanged at the end of the period.

Credit risk refers to the decline in the value of a swap or other derivative if there is a decline in the credit quality of one of the parties to the derivative. Ultimately, the concern is that the increasingly risky party will default on its obligations Default risk is a function of both the creditworthiness of a party and the fact that interest rates or exchange rates have changed since the swap's origination date so that the swap is now a liability for that party. Credit derivatives are used to manage credit risk and default risk. In a credit swap, the buyer receives a periodic cash inflow only if there has been some triggering credit event.

Reference

Patel, Navroz. 2001. "Credit Derivatives: Vanilla Volumes Challenged." *RISK*, Vol. 14, No. 2, February, pp. 32–34.

Notes

[1]In some swaps, the floating-rate payment is based on the floating rate that exists two days before the payment date. These are called **arrears reset swaps, arrears swaps, reset swaps, or in-arrears swaps.**

[2]In practice, several different methods are used to calculate partial year cash flows, and they are described in Appendix B to Chapter 5. The 30/360-day basis essentially assumes there are 12 months in a year, each of which has 30 days. An 8% interest rate becomes a 2% quarterly rate, or a 4% semiannual rate. A few exceptions exist, but they are unimportant for the purposes of this text.

[3]The example is simplified by assuming a 30/360 basis for computing the floating-rate payment. In practice, this is not typical. See note 2.

[4]Note that the numerator of the settlement amount equation is $60,000. This is a future amount, to be received six months hence. The denominator converts the $60,000 to be received six months hence into a present value of $57,887.12.

[5]Several Treasury notes may exist with similar maturities about equal to the swap's tenor, each of which has a different coupon rate. On the swap's origination date, it is specified exactly which of these notes' yields the quoted spread will be applied against.

[6]To learn more about Euribor, access www.euribor.org.

[7]This will be illustrated when swap valuation principles are discussed in Chapter 12.

[8]In an **index-accreting swap**, the notional principal *increases* at one or more times before the swap's maturity date.

[9]The term "typically" is used because, as with all OTC derivative contracts, anything is negotiable.

[10]This spread, in part, reflects the difference in interest rates for Treasury securities of the same maturity as the swap. For example, two-year riskless Japanese interest rate might be 2%, and the two-year riskless U.S. interest rate might be 6%; hence the example incorporates a 400-basis-point spread.

[11]In some commodity swaps, two commodities serve as the notional principals. The fixed price applied to the notional principal of one commodity is exchanged for the floating price applied to the notional principal of a different commodity. Floating–floating commodity swaps, with two different commodities also exist (these are basis swaps). Also, some commodity swaps may compute the average floating price during the period as the floating price for the swap (this is similar in concept to an Asian option, in which the payoff is based on the average price of the underlying asset during a defined period of time, rather than its expiration day price).

[12]Credit risk refers to the risk that the deterioration in the creditworthiness of a party will reduce the probability that the more risky party will make future payments. This reduces the value of the swap for the more creditworthy party. Default risk is the risk that a payment will not be paid.

[13]Changes in the floating interest rate also affect the incentive to default. In the example of Section 11.2.2, a decline in U.S. interest rates will add to the pay-fixed party's incentive to default, since he is paying yen fixed and receiving floating dollars. The decline in the interest rate at which the floating dollars are received can be invested to further reduce the asset value of the swap for the pay-fixed party.

[14]Note that in this example, the mutual fund will be paid off if BBB's creditworthiness declines or if there is an increase in the market's aversion to default risk. The latter will affect all corporate bonds exposed to nontrivial default risk, even if there is no real change in the credit quality of a specific bond issuer.

[15]See Patel (2001).

PROBLEMS

11.1 In an arrears reset swap, the floating payment is based on LIBOR on the payment date. In a normal fixed–floating swap, the floating payment is based on LIBOR one full period before the actual payment date. Suppose that you are a receive-fixed party, and you believe that the floating interest rate will decline. The swap dealer is quoting you the same fixed rate for both types of swap. All else equal, would you prefer to enter into an arrears reset swap or a normal swap? why?

11.2 Consider a three-year plain vanilla fixed–floating interest rate swap. The notional principal is $20 million. The swap fixed rate is 6%. The floating rate is six-month LIBOR. The payment dates are every six months, beginning six months hence. On the origination date, six-month LIBOR is 5.5%. The day count basis for both the fixed rate and the floating rate is 30/360. On subsequent dates, the six-month. LIBOR is:

Time	6-Month LIBOR
0.5	5.25%
1.0	5.5%
1.5	6%
2.0	6.2%
2.5	5.44%

Compute the cash flows that are exchanged between the two counterparties.

11.3 Suppose the fixed rate payer in Problem 11.2 buys a 12×18 FRA with $NP = \$20$ million, and a contract rate of 6% (buying a FRA is an agreement to buy LIBOR). What is the settlement day cash flow? Note that Problem 11.2 presents the subsequent six-month LIBORs. Assume a 30/360 day count method. Why does your answer differ from the time 1.5 cash flow you computed for the swap in Problem 11.2?

11.4 You observe a market maker quoting a bid price of 6.8% to firm A on a fixed–floating interest rate swap and a bid price of 7.0% on the same swap to firm B. Suppose you also observe that the asked quotes are greater than the bid quotes for each counterparty. Is the market maker going to become the pay-fixed or receive-fixed counterparty in these swaps? Why? Which firm, A or B, is financially weaker? Why?

11.5 If interest rates decline after a fixed–floating interest rate swap has been originated, who benefits, the pay-fixed counterparty or the receive-fixed counterparty? Equivalently, for which of the two counterparties does the swap contract have positive value after the interest rate decline? After the interest rate decline, who has the greater probability of defaulting?

11.6 What changes do swap dealers make to swap contracts when dealing with counterparties that are perceived to be likely to default?

11.7 On a fixed–floating interest rate swap, a dealer quotes a swap spread price of 35/32. What does this mean?

11.8 A firm observes that 10-year Treasury yields are at 7.1%, and two-year Treasury notes are yielding 7.0%. It believes that the 10-basis-point spread is very narrow and will likely widen in the near future. What kind of swap can the firm enter into speculate on these beliefs?

11.9 A firm wants to enter into a two-year fixed–fixed yen–deutschemark (DEM) currency swap. The spot exchange rate is ¥77/DEM. The principal amount is 20 million DEM. The swap dealer is quoting fixed rates on this type of swap of 2% in yen and 5% in DEM. Payments are quarterly, on a 30/360 day count basis. The firm wants to pay fixed DEM and receive fixed yen. What are the cash flows that will be exchanged if the firm agrees to the swap?

11.10 A firm wants to enter into a two-year currency swap. The firm wishes to pay a fixed rate of 6% in euros and receive floating sterling (British pounds). The euro payments will be semiannual, and the pound payments will be quarterly, both on a 30/360 day count basis. The principal amounts are £40 million and €70 million. Today, three-month sterling LIBOR (the floating rate) is 5%. Subsequent realizations of three-month sterling LIBOR are as follows:

Time	3-Month Sterling LIBOR
0.25	5.25%
0.50	6%
0.75	6.3%
1.0	6.85%
1.25	6.5%
1.50	6.2%
1.75	6%
2.0	6.3%

What are the cash flows that the firm will pay and receive, at each date?

11.11 A firm enters into a diff swap in which it agrees to pay interest in euros based on three-month dollar LIBOR and receive interest in euros based on three-month Euribor. The notional principal is €200 million. A margin of 50 basis points will be added to three-month Euribor. Payments are quarterly. The rates that determine the first payment are 6% (Euribor) and 5.65% (U.S. dollar LIBOR). The initial exchange rate is $0.70/€. What is the amount of the first difference check? Why is a margin of 50 basis points added to Euribor?

11.12 A gold mining firm and a gold user wish to enter into a commodity swap, with a swap dealer as an intermediary. The gold producer wants to fix the price it will receive for the gold it will mine over the next three years. The gold user wants to fix the price it will have to pay for the gold it needs for the next three years. The notional principal is 10,000 oz. of gold. The fixed price is $420/oz. Settlement is semiannual, based on the average price of gold during the past six months. Subsequent spot gold prices are as follows:

Time	Average Gold Price During Past Period
0.5	$405
1.0	$430
1.5	$468
2.0	$502
2.5	$448
3.0	$400

Determine the cash flows for the gold producer.

11.13 When will a firm exercise its swaption that gives it the right to enter into a swap as a fixed rate payer: when interest rates have risen, or when they have declined? When will a firm exercise the option in its puttable swap to exit an existing swap in which it is the fixed-rate receiver: when interest rates have risen, or when they have declined?

11.14 (a) What is a credit derivative? (b) How might a credit swap be used by a mutual fund that invests in bonds that are rated below investment grade?

11.15 In words, explain and interpret the "plain vanilla" interest rate swap diagram in Figure P11.15. Party A is the pay-fixed party. The price of the swap is 8%. The floating rate for the swap is LIBOR. LIBOR on the swap initiation day is 6%. LIBOR six months later will be 7% (Fed Chairman Alan Greenspan has told you this). Swap payments are made semi-annually. The notional principal of the swap is $35 million.

As part of your answer, you should (a) explain how the swap works, (b) label the arrows that define the swap, (c) compute the first net payment (be sure to state which party pays what amount), (d) explain why party A is entering into this swap, and (e) explain anything else you believe is relevant for this question, given the information provided.

11.16 A firm is part of a "plain vanilla", typical, fixed–fixed currency swap. It is paying yen at a fixed rate of 2% and receiving dollars at a fixed rate of 6%. For this firm, the swap is essentially equivalent to (select either **a** or **b**)

a. having a yen-denominated asset and a dollar-denominated liability

b. having a dollar-denominated asset and a yen-denominated liability

11.17 In addition to what you were told in Problem 11.16, you know that the principal amount of the swap is $30 million, and the spot exchange rate on the day the swap was initiated was ¥105/$. Alan Greenspan has told you that he guarantees that on the swap's termination day, the spot exchange rate will be ¥110/$. There are two years remaining to the swap, and there will be an exchange of principal amounts on the swap's termination date. Payments are made semiannually. Therefore, which of the following is true about the firm's last swap payment (on the swap's termination day):

a. The firm will pay ¥288,571.

b. The firm will pay ¥300,000.

c. The firm will pay ¥30,300,000.

d. The firm will pay ¥31,500,000.

e. The firm will pay ¥3,181,500,000.

f. The firm will pay ¥3,333,000,000.

g. The firm will have to pay both yen and $30 million dollars on the termination day.

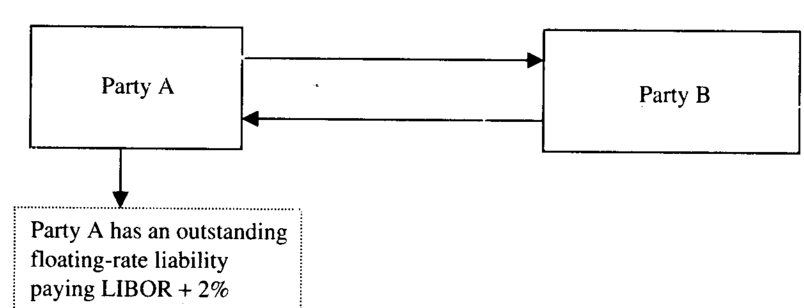

Figure P11.15

CHAPTER 12

Using Swaps to Manage Risk

Swaps are an efficient vehicle for firms to manage the composition of many assets and liabilities. Because the value of an asset or liability is the present value of cash flows that are attributed to that asset or liability, it becomes apparent that swaps also let firms manage revenues and expenses. Swaps are preferred to forwards and futures under the following circumstances:

a. When the cash flow stream is periodic, or regularly occurring. A single risk exposure is better hedged using forwards or futures.

b. When the cash amounts are equal and occur at regular (e.g., quarterly) intervals.

c. When the user wishes to manage the entire stream of cash flows in the same way. In contrast, forwards and futures allow the risk manager to hedge a portion of some cash flows, leave others unhedged, and to hedge others completely.

In this chapter, the focus is on using swaps to manage the risk to the firm caused by an unexpected financial price change. However, swaps can be used to create value for a firm. Using interest rate swaps to reduce borrowing costs is one example of how a swap can create value for a firm.

12.1 Using Interest Rate Swaps

12.1.1 Swapping to Lower Borrowing Costs

Frequently, the default risk premium on issued debt instruments is greater in the long-term, fixed-rate bond market than it is in the floating-rate debt market. That is, there is a quality differential between fixed and floating borrowing. In the early days of swaps, market participants attributed the quality differential to the relatively risky, low-rated firm having a **comparative advantage** in the floating-rate market.[1]

Suppose a quality differential exists between fixed and floating borrowing. Consider the following example. The BBB Company, rated BBB by Standard & Poor's, has the opportunity to borrow either at a fixed rate of 8% or at a floating rate of LIBOR + 75 basis points.[2] AA Corporation, which is rated AA by Standard & Poor's, can borrow either in the long-term fixed-coupon debt market at 7% or at a floating interest rate of LIBOR + 25 basis points.

The AA Corp. has an absolute advantage in both markets because it faces lower interest rates in both markets. But AA has a comparative, or relative, advantage in the fixed-rate market because it can borrow there at 100 basis points below BBB Co. In contrast, BBB has a comparative advantage in the floating-rate market because it must pay only 50 basis points more than AA in the floating-rate market. In the fixed-rate market, BBB must pay 100 basis points more than AA. The term "quality differential" describes the differences in credit risk spreads that the two firms face in the two different markets (the fixed market and the floating-rate market).

Assume that neither firm wants to actually borrow funds in the market in which it enjoys its comparative advantage. In other words, BBB wants fixed-rate debt, and AA wants floating-rate debt.

A plain vanilla fixed-for-floating interest rate swap can benefit both firms when such a comparative advantage exists. First, each party should borrow in the market in which it has a comparative advantage. Then, the two parties can engage in an interest rate swap. The result will be that each firm will have effectively borrowed in the market in which it wants to borrow and will have done so at a lower cost than would have been possible without the swap.

Thus, AA goes to the long-term fixed-rate bond market and issues \$100 million in seven-year fixed-rate debt with a coupon of 7%. BBB borrows the same amount in the floating-rate debt market for seven years at a rate of six-month LIBOR + 75 basis points. Payments for both firms will be semiannual.

Then, the two firms enter into a plain vanilla fixed-for-floating interest rate swap with a tenor of seven years. AA becomes the receive-fixed party in the swap. BBB is the pay-fixed party. AA agrees to pay a floating rate of LIBOR, and receive a fixed rate of 7%. Counterparty BBB agrees to pay a fixed rate of 7%, and receive a floating rate of LIBOR. The entire transaction is illustrated in Figure 12.1.[3] To summarize:

AA Corp.

Pays 7% to the capital market	−7%
Receives 7% fixed in the swap	+7%
Pays LIBOR in the swap	−LIBOR
Net	−LIBOR (floating)

If AA Corp. had issued a floating-rate bond directly to the capital market, it would have had to pay LIBOR + 25 basis points. Therefore, AA has saved 25 basis points by issuing **synthetic floating-rate debt** with the aid of the swap. That is, by issuing fixed-rate debt and then swapping cash flows, AA creates **synthetic floating-rate debt.**

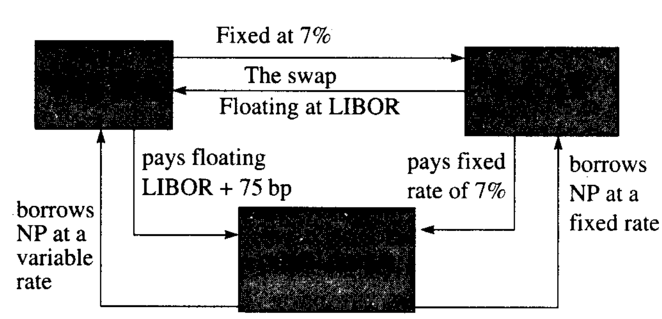

Figure 12.1 How swaps can lower the borrowing costs for both counterparties. After the swap, party BBB has borrowed at a fixed rate of 7.75%, and party AA has borrowed at a floating rate equal to LIBOR. By swapping, each party has saved 25 basis points (bp) in comparison to the situation in which each issues in its desired market (which is *not* the market in which it has a comparative advantage).

BBB Company

Pays LIBOR + 75 bp to the capital market	−LIBOR − 75 bp
Pays 7% fixed in the swap	−7%
Receives LIBOR in the swap	+LIBOR
Net	−7.75% (fixed)

If BBB had issued a fixed-rate bond directly in the spot debt market, it would have had to pay 8%, so BBB has saved 25 basis points by issuing **synthetic fixed-rate debt** with the aid of the swap.

The gains derivable from these swaps that lower borrowing costs have diminished in recent years, but judging from surveys on derivatives usage, the gains still exist.[4]

It is important to note that these swaps are contracted with swap dealers. That is, firms do not arrange swaps themselves as in the foregoing example. Thus, in reality, each party will not save 25 basis points. Firm AA might receive 6.97% fixed and pay LIBOR. Firm BBB might pay 7.03% and receive LIBOR. The counterparty to each firm would be a swap dealer. The swap dealer would be neutral regarding LIBOR but earn a profit of 6 basis points (7.03% − 6.97%) per year on the notional principal of the swap. Thus, in this example, each firm would only save 22 basis points because of the swap.

Put another way, there is a total benefit of 50 basis points to be carved up between AA, BBB, and the intermediary (the swap dealer). To compute the number of total number of basis points that can be saved, subtract the rate differential in the floating-rate market from the rate differential in the fixed-rate market:

Rate differential in the fixed-rate market	8% − 7% = 100 basis points
Rate differential in the floating-rate market	(LIBOR + 75 bp) − (LIBOR + 25 bp) = 50 basis points
Number of basis points to be saved	50 basis points

Frequently, however, after the expenses, the fees, and the swap dealer's bid–asked spread have been accounted for, the comparative advantage will be too small for any net benefit to be realized by the two counterparties.

It is important to note that these firms could have also used a strip of futures or a strip of FRAs to achieve essentially the same result as the swap. However, the advantages of the swap market over the futures market (in particular) are many. For example:

1. There are no six-month LIBOR futures contracts. Two three-month Eurodollar futures contracts with adjacent delivery dates would have to be traded to effectively create a six-month LIBOR contract.
2. One swap transaction will cover the firms for seven years.
3. If each firm needs $100 million, the transaction costs of using futures would be high. Each three-month LIBOR contract covers $1 million. Thus, 100 contracts would cover a $100 million bond issue over a single three-month period; 200 contracts would be needed for a six-month period. This means that over the seven-year tenor, 2800 contracts would have to be traded by each firm.
4. AA, being a high quality firm, could negotiate an advantage to itself, namely, that its swap not be marked to market. BBB is riskier, so it may have to offer collateral to the swap

dealer, or the swap dealer may demand that the swap be marked to market. Both firms would be marked to market if futures were used.

5. Eurodollar futures contracts expire quarterly. Basis risk would exist unless a bond payment date coincided with the futures expiration date.

The primary difference of a swap versus a strip of FRAs is that each FRA will have a different price. The benefits that can be realized from lower borrowing costs should be about equal for the strip of FRAs and for the swap. In general, a FRA is used to hedge the borrowing or lending rate for a single period, while a swap is used to hedge a series of periodic cash flows. However, the costs of engaging in each transaction, and the prices of the contracts, should be carefully analyzed to decide which hedging vehicle is better.

12.1.2 Swapping to Hedge Against the Risk of Rising Interest Rates

Swaps are valuable hedging tools because they allow a firm with a sizable floating-rate debt to hedge, or protect, itself against the risk of rising interest rates. If interest rates do rise, a firm with floating-rate debt will experience higher floating interest rates. Because interest paid is an expense item on an income statement, net income declines if interest expense rises, all else equal. A greater percentage of the firm's operating income will flow to the floating-rate bondholders, leaving less for the stockholders. Thus, firms with floating-rate debt are exposed to the risk of rising interest rates.

Bodnar, Hayt, and Marston (1998) report that 96% of firms in their survey at least sometimes swap from floating rates to fixed rates, whereas only 60% of firms swap from fixed to floating (obviously, many firms do both). These result could be attributed to the very low level of interest rates that existed at the time of the survey, in 1998; firms would want to lock in those low rates by transforming their floating-rate debt to fixed-rate debt via the swap market.

Entering into a fixed-for-floating interest rate swap as the pay-fixed party permits a firm to transform existing floating-rate debt into synthetic fixed-rate debt. That is, the firm can lock in a fixed rate for the remaining life of the bond.

Before the swap, the firm has floating-rate debt. As Figure 12.2 illustrates, after the swap, the firm has altered the nature of this liability and is now in the position of having a fixed-rate liability.

Consider the following example of transforming a floating-rate asset into a fixed-rate asset. Suppose a Japanese bank has outstanding a considerable amount of three-month certificates of deposits (CDs), and these CDs are repriced at a new interest rate every three months. The bank

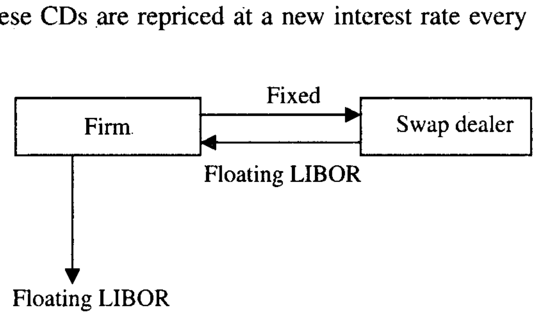

Figure 12.2 A swap changes a floating-rate liability into a synthetic fixed-rate liability, and hedges the firm against rising interest rates.

fears that interest rates will rise, and it would like to lock in its funding costs for a longer period of time. Interest on these CDs is paid at the maturity of the CD, and the interest rate is set when the customer buys the CD. The Japanese bank would like to have a greater amount of fixed rate debt. It can enter into a fixed-for-floating interest rate swap in which it is the pay-fixed party.

Suppose that on the CDs, the bank pays interest at the rate of yen LIBOR+25 basis points. The desired swap tenor is three years, with quarterly payments. The swap dealer quotes a fixed rate of 2% in exchange for yen LIBOR. The notional principal amount is ¥3 billion.

This transaction locks in a fixed rate 2.25% for three years, as illustrated in Figure 12.3.

Table 12.1 shows that regardless of what happens to interest rates in the future, the Japanese bank will pay a fixed rate of 2.25%. The column heads show three different interest rate levels (in terms of three-month yen LIBOR) that occur at time t. Below that are the interest paid at time $t+1$ on ¥3 billion in CDs, and the swap cash flows on a notional principal of ¥3 billion.

12.1.2.1 Forward Swaps

A **forward swap** can be used to protect against a rise in interest rates in the future. A forward swap is a forward contract that obligates the firm to enter into a swap some time in the future.

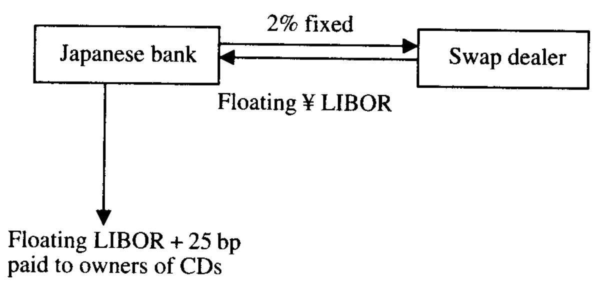

Figure 12.3 The Japanese bank is the pay-fixed party in this swap. It uses the swap to hedge against rising interest rates.

TABLE 12.1 A Swap in Which the Pay-Fixed Party Has Locked in a Fixed Rate of 2.25% Regardless of the Floating Rate

	Three-Month Yen LIBOR at Time t		
Payment	**1.82%**	**2%**	**2.05%**
CD interest paid at time $t+1$	−15,525,000[1]	−16,875,000	−17,250,000
Swap cash flow payment at time $t+1$	−15,000,000[2]	−15,000,000	−15,000,000
Swap cash flow receipt at time $t+1$	+13,650,000[3]	+15,000,000	+15,375,000
Net payment	−16,875,000[4]	−16,875,000	−16,875,000

[1] 1.82% + 25 bp = 0.0207. The quarterly payment is then (0.0207/4)(¥3 billion) = ¥15,525,000.
[2] Fixed swap payment of 2%. The quarterly payment is then (0.02/4)(¥3 billion) = ¥15 million.
[3] Floating swap payment is LIBOR. The quarterly payment is then (0.0182/4)(¥3 billion) = ¥13,650,000.
[4] A quarterly payment of ¥16,875,000 is an annual interest rate of 2.25%: (¥16,875,000 × 4)/¥3 billion = 2.25%.

Usually, the initial payments that will be exchanged in forward swaps commence one year or more after such an agreement has been originated.

Several scenarios would dictate the use of a forward swap to manage risk exposure:

a. To hedge against an anticipated rise in interest rates next year, a firm would enter into a forward swap as a fixed-rate payer. If interest rates do rise, the floating-rate payments the firm will receive will be greater than its fixed payments.

b. A firm wants to have a floating-rate liability next year. However, because it will likely have a comparative advantage in issuing fixed-rate debt, it instead plans on issuing long-term fixed-rate debt next year. This firm can enter into a forward swap as the receive-fixed party. One year hence, it will issue fixed-rate debt; at that time, the swap's obligations will commence, and the firm that wanted floating-rate liability at that time will indeed have synthetic floating-rate debt. This example would apply if the firm expected interest rates to fall next year.

c. Finally, consider a firm that already has floating-rate debt outstanding and anticipates that rates will stay low for about a year before beginning to rise. It should enter into a forward swap as the pay-fixed party.

Note that elements of speculation are associated with the foregoing scenarios. The firm in example **c** has outstanding floating-rate debt and is exposed to the risk of rising interest rates. To hedge, it should lock in its interest rate expense today, and the way to do that is to enter into a swap that begins today, not a year from today. Entering into a forward swap results in continued exposure to rising interest rate risk for the next year. Of course, the forward swap strategy will work only if the firm's beliefs about interest rates during the next year are correct.

Bodnar, Hayt, and Marston (1998) report that 66% of the firms in their sample timed their interest rate hedges based on their view of what interest rates would do (rise or fall) in the future. In other words, they practiced selective hedging.

12.1.3 Swapping to Hedge Against the Risk of Falling Interest Rates

Consider the case of a mutual fund manager who just added some rather unique corporate notes to his fixed-income portfolio. Suppose a recent sharp run-up in interest rates made these corporate notes particularly attractive to the fund manager, who paid slightly under par for the notes. However, the fund manager now fears a reversal in interest rates. Of course, this would ruin the attractive return the notes currently offer because of an interesting feature of these corporate notes. The coupon notes in question have a face value of $750 million and a variable coupon, currently at 8%. Starting today and every six months hereafter for five years, the variable coupon rate is set equal to the spot 10-year U.S. Treasury note rate plus 150 basis points.

The fund manager decides to enter an interest rate swap wherein he is the receive-fixed party. A swap dealer has agreed to pay the fund manager a fixed rate of 7.96% for each of the 10 remaining coupon payments in exchange for the variable coupon payment with the notional value set to $750 million. Given the series of subsequent spot U.S. T-note rates listed in Table 12.2, we can compute the resulting cash flows.

TABLE 12.2 Computing Cash Flows That Would Result from Various T-Note Rates

Time	Spot U.S. 10-Year T-Note Rate	Payment to Swap Dealer	Receipt from Swap Dealer	Net Cash In flow (Outflow)
Now	6.50%	$0	$0	$0
Months later				
6	6.25%	$30,000,000[1]	$29,850,000[2]	$(150,000)
12	6.00%	$29,062,500	$29,850,000	$787,500
18	6.25%	$28,125,000	$29,850,000	$1,725,000
24	6.35%	$29,062,500	$29,850,000	$787,500
30	6.75%	$29,437,500	$29,850,000	$412,500
36	7.00%	$30,937,500	$29,850,000	$(1,087,500)
42	7.25%	$31,875,000	$29,850,000	$(2,025,000)
48	7.00%	$32,812,500	$29,850,000	$(2,962,500)
54	6.95%	$31,875,000	$29,850,000	$(2,025,000)
60		$31,687,500	$29,850,000	$(1,837,500)

[1] $[(0.065 + 0.015)/2]\$750,000,000 = \$30,000,000$.
[2] $[0.0796/2]\$750,000,000 = \$29,850,000$.

In this example, the fund manager was able to hedge the risk of a decrease in interest rates over the short run. However, because interest rates rose during the later years of the swap, the fund manager would have lost money in this period (assuming nothing else was done to manage the portfolio after once the swap had been entered).

Consider another example in which an investments company can invest only in assets with maturities less than a year. The existing yield curve is sharply upward sloping, and two-year debt instruments are yielding a great deal more than one-year securities. Moreover, the company believes that interest rates are about to decline.

This investments company can synthetically extend the maturity of investments by engaging in a swap as the receive-fixed party. It can agree to pay the one-year floating interest rate (perhaps the Treasury rate) and receive the existing, fixed, two-year interest rate. No physical securities need to be traded, and the investments company still owns only securities with maturities less than one year.

12.1.4 Swaps, Gap Analysis, and Duration

Financial institutions, such as banks, utilize the tools of **gap analysis** and **duration** to analyze their exposure to interest rate risk:

$$\text{gap} = \text{rate} - \text{sensitive assets} - \text{rate sensitive liabilities}$$

An asset or liability is "rate sensitive" when it will be repriced during a defined period. The defined period is called the **gapping period**. For example, if a bank owns a debt instrument with a maturity of five months, or has lent money to a client and the loan is due to be repaid in five months, the face value of that security or loan would be a rate-sensitive asset for any gapping period greater than five months. The security will have to be replaced, probably with a higher or lower interest rate, in five months. If a bank has issued a three-month CD, it is a rate-sensitive liability for any gapping period greater than three months.[5]

If gap is positive, the bank has more rate-sensitive assets than liabilities (for the relevant gapping period). This means that if interest rates decline, more assets will be repriced at lower rates, reducing net income. If gap is negative, the bank faces tne risk of rising interest rates because more liabilities will be repriced at higher rates, increasing interest expense.

Financial institutions (and other firms, too) also estimate the duration of their assets and liabilities. One definition of duration (dur) is:

$$\text{dur} = -\frac{\%\Delta\text{value}}{\%\Delta(1+r)} = -\frac{(V_1 - V_0)/V_0}{(r_1 - r_0)/(1+r_0)}$$

where

V_0 = today's value (of an asset or liability) at an initial interest rate of r_0,

V_1 = the value at a new interest rate of r_1.

Rearranged, this is

$$\%\Delta\text{value} = -\text{dur} \times \%\Delta(1+r)$$

If the duration of a financial institution's assets exceeds the duration of its liabilities, a rise in interest rates will cause the value of its assets to decline more than the resulting decline in the value of its liabilities. Then, the value of owner's equity must decline. Figure 12.4 uses a simple balance sheet to depict this situation.

If the duration of the institution's liabilities is greater than the duration of its assets, the owners face the risk of declining interest rates. Should rates decline, the increase in the value of the liabilities will exceed the increase in the value of the assets. This also means that the value of owner's equity will decline. To summarize, if:

duration assets > duration liabilities
&/or
negative gap
} risk is that interest rates will rise.

duration assets < duration liabilities
&/or
positive gap
} risk is that interest rates will fall.

A financial institution can use swaps to hedge against the risk of changing interest rates when the duration of its assets differs from the duration of its liabilities and/or the financial institution has a positive or negative gap.

If the institution is exposed to the risk of rising interest rates, it should be the pay-fixed party in a swap. If interest rates subsequently rise, the floating rate that it receives will rise. Thus, the

Assets	Liabilities
Long duration => if r rises, V declines a lot	Short duration => if r rises, V declines by a smaller amount
	Owner's Equity
	∴ the value of owners equity must decline, since equity = assets − liabilities

Figure 12.4 If the duration of a firm's asset exceeds the duration of its liabilities, the firm is exposed to the risk that interest rates will rise. A balance sheet approach illustrates this concept.

swap benefits will offset its other exposures to rising interest rate risk. An institution that is exposed to the risk of falling interest rates should be the receive-fixed party in a swap.

12.2 Using Currency Swaps

12.2.1 Swapping to Lower Borrowing Costs in a Foreign Country

Multinational firms borrow funds in many different currencies to finance their international operations. A typical multinational will have a comparative advantage in borrowing in some countries, and not in others. Comparative advantages will arise when a firm that is very well known in one country is not well known in another. It may then face relatively lower borrowing costs in the country in which it is well known. Oddly enough, a multinational may also face relatively lower borrowing costs in a country in which it had never before floated a bond issue. That country's investors may be eager to realize the diversification benefits of buying bonds, issued in their own country's currency, sold by a large foreign firm. Should that country's economy sour, its domestic firms may experience higher default rates. But at the same time, the foreign company may continue to prosper and pay its debts. Thus, to realize the benefits of diversification, it is beneficial to buy bonds issued in your home country's currency, but issued by a wide range of issuers, including domestic firms, foreign firms, and governments.

A firm or government that has a comparative advantage in issuing bonds in currency A, but needs funds denominated in currency B, can exploit this situation as follows. First, debt is issued in currency A, and then a currency swap is entered in which currency A is received and currency B is paid. As a result, a lower borrowing cost in currency B may be realized than if it had directly issued bonds denominated in currency B.

As an example, suppose that Phyllis Morris is a large U.S.-based multinational that has never before issued bonds denominated in Swedish krona (SEK). Swedish investors are eager to buy SEK-denominated bonds issued by this highly rated U.S. company. But, Phyllis Morris needs British pounds (GBP). If Phyllis Morris issues bonds in Great Britain, denominated in GBP, it will have to pay 9%. If Phyllis Morris floats a bond issue in Sweden, denominated in SEK, it will face an interest of 10%. The company approaches a swap dealer and requests a quote on a fixed-for-fixed currency swap in which Phyllis Morris will receive SEK at a fixed rate of 10%. The swap dealer quotes a price of 8.7% fixed in terms of GBP.

Figure 12.5 shows that if Phyllis Morris issues the SEK-denominated debt, which is the currency in which it has a comparative advantage but is also the currency it does not want, and also engages in the fixed-for-fixed currency swap, it will end up with synthetic GBP debt and realize a savings of 30 basis points. Recall that there are three stages to a typical currency swap. The principal amounts, denominated in two different currencies, are exchanged both at origination and at maturity. The amounts that are swapped are equal in value at origination, since they are linked by the exchange rate at origination. In this example, the spot exchange rate at origination is SEK13.91/£. Phyllis Morris wishes to borrow £15 million, so the face value of its bond issue will be SEK208.65 million. At both origination (Figure 12.5a) and maturity (Figure 12.5c), £15 million will be exchanged for SEK208.65 million.

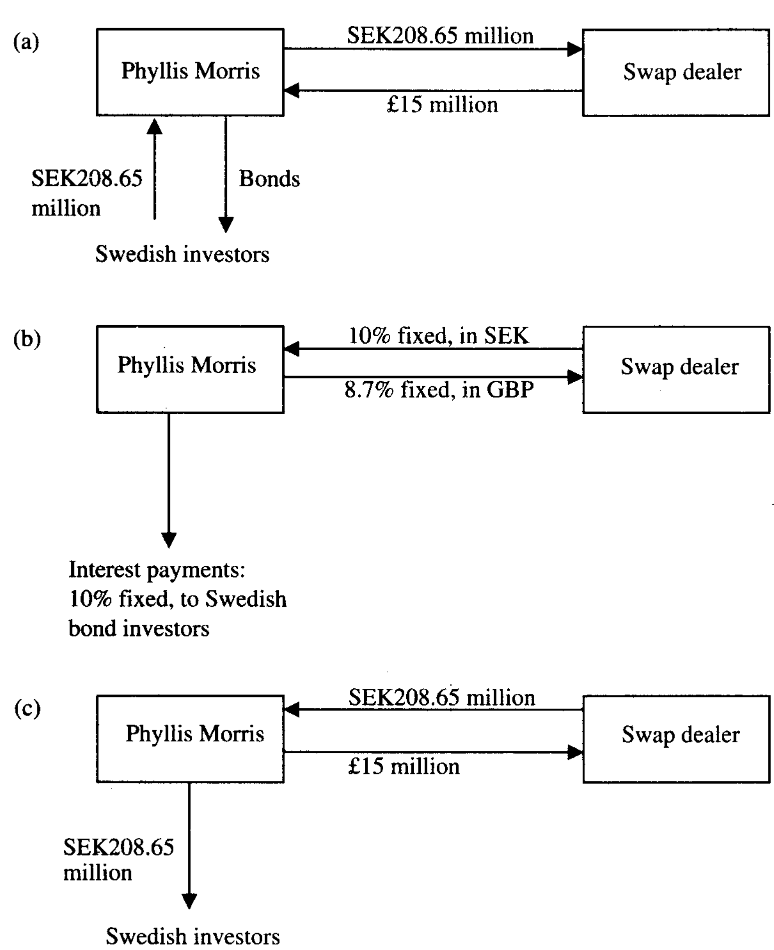

Figure 12.5 Cash flows (a) at origination of swap, (b) during the swap (interest payments are exchanged), and (c) at maturity of swap.

12.2.2 Swapping to Hedge Against the Risk of a Decline in a Revenue Stream

Consider U.S. Apple, a U.S.-based firm that exports apples and sells them for yen in Japan. An abbreviated form of its income statement is:

Revenues = $P\ Q$
– Expenses
Operating income

where P is the price the firm receives for the apples it sells in Japan (¥), Q is the quantity of apples it sells in Japan, and expenses are in U.S. dollars.

The goal of U.S. Apple is maximize its dollar profits—typical for a U.S.-based firm. U.S. Apple is exposed to the risk that the $/¥ exchange rate will fall. If the $/¥ declines, the dollar value of the firm's yen revenues will be less, and its dollar profits will be less.

U.S. Apple can use a fixed-for-fixed currency swap to hedge its risk exposure. It can estimate its yen-denominated revenues for the next several years, and agree to pay fixed yen and receive fixed U.S. dollars in each of the next several years.

U.S. Apple will still be exposed to the risk of fluctuation in the quantity of apples it sells in Japan. The number of apples it can sell in Japan will vary as its crop size (in the United States) varies, as the selling price of apples grown and sold in Japan varies, as the prices of other competing fruits in Japan varies, as import/export laws change, and as tastes change in Japan.

12.2.3 Swapping to Hedge Against the Risk of an Increase in Cost

Chocoswiss is a Swiss manufacturer of liqueur-filled chocolates. It must import all its liqueurs from France, and it pays for the liqueurs in euros. However, it sells its product in Switzerland only. Chocoswiss want to maximize its profits, which are denominated in Swiss francs (SFR). An abbreviated version of Chocoswiss' income statement is:

Revenues (in SFR)
– Expenses (a significant portion is in euros)
Operating income

Chocoswiss faces the risk that the SFR/€ will rise. If the SFR/€ rate rises, then the SFR cost of its imports will rise. As costs rise (denominated in SFR), SFR-denominated profits for Chocoswiss will decline. To hedge its currency risk exposure, Chocswiss can use a fixed-for-fixed currency swap in which it pays SFR and receives euros. There is no need to exchange principal amounts.

12.2.4 Swapping to Hedge Against the Risk of a Decline in the Value of an Asset

Suppose a U.S. company has a three-year £50 million investment (an asset) that yields 7% annually (in GBP) and pays interest twice per year. The current exchange rate is $1.60/£. The U.S. corporate treasurer thinks that the dollar will strengthen against the pound sterling. Equivalently, this means that the dollar value of the GBP will decline (the \$/£ exchange rate will decline), which means that the dollar value of any GBP-denominated assets will decline.

If the treasurer is correct, each future interest inflow of £1,750,000 will purchase less than \$2,800,000. For example, if the exchange rate is \$1.50/£, the interest payment of £1,750,000 will purchase only \$2,625,000. However, because the current three-year interest rate in the United States is 7.40%, the treasurer does not want to swap each subsequent interest payment of £1,750,000 for only \$2,800,000. Not only will the decline in the value of the GBP mean that the value of the interest rates will be less, but the dollar value of the U.S. firm's investment will decline, too.

The treasurer finds a swap dealer willing to swap interest payments each six months of £1,750,000 for \$2,940,000 over the next three years. In addition, there will be a final swap of £50 million for \$80 million. Under this swap, the U.S. company has transformed its three-year £50 million investment that yields 7% into a three-year \$80 million investment that yields 7.35%.

As another example, consider a Japanese company that owns some real estate in the United States; that is, the Japanese company has a dollar-denominated asset. If the ¥/\$ exchange rate declines, the value of this asset, in yen, will decline. To hedge, the Japanese company can buy yen futures or forwards. Alternatively, the Japanese company can enter into a swap, paying dollars and receiving yen.

12.2.5 Swapping to Hedge Against the Risk of a Rise in the Value of Liability

If the value of a firm's liability rises and its asset values remain unchanged, it follows that the value of the firm's stock must decline. This must be the case because assets = liabilities + owners' equity.

Suppose a U.S. company has a two-year debt (a liability) of €100,000,000 at 7.7% annually and interest is paid quarterly. The current exchange rate is €0.9720/\$. The U.S. corporate treasurer's staff is predicting that the dollar will weaken against the euro (i.e., the €/\$ exchange rate will fall). This is equivalent to predicting that the \$/€ rate will rise. If the dollar price of the euro rises, then the dollar-denominated value of this firm's liability will rise.

If the staff is correct, each future interest payment of €1,925,000 will cost more than \$1,980,453. For example, if the exchange rate changes to €0.9400/\$, the interest payment of €1,925,000 will cost the firm \$2,047,872.

The treasurer finds a swap dealer willing to swap quarterly cash flows of €1,925,000 for \$2,004,750 over the next two years. In addition, there will be a final swap of €100,000,000 for \$102,880,658. Under this swap, the U.S. company has transformed its two-year 7.7% debt for €100,000,000 into a 2-year \$102,880,658 debt with an interest rate of 7.79%.

12.3 Using Commodity Swaps

A commodity swap can be used to fix the price of any physical commodity over a time period. Commodity swaps can be arranged through a swap deal for commodities such as aluminum, copper, silver, gold, crude oil, heating oil, or any other desired commodity.

In a commodity swap, the fixed-price payer makes periodic fixed payments for a notional quantity of the specified commodity; this locks in the purchase price for the commodity. The floating-price payer makes periodic payments for a notional quantity based (usually) on an average of the spot price over some time period. The notional quantity of the asset is not exchanged between the counterparties to the commodity swap. Instead, each party makes or takes delivery of the commodity through the spot market.

To demonstrate how a commodity swap can be used, consider the following. Suppose a small airline wishes to fix the price it pays for its monthly use of 50,000 gallons of jet fuel. The airline chooses a three-year tenor. At the same time, a small oil refiner wishes to fix a three-year price it receives for its monthly production of 100,000 gallons of heating oil (which is often used to price jet fuel). Note that it is not necessary for these two parties to arrange a direct swap: each can use a swap dealer. The terms of the swap call for a monthly floating payment based on the average of the daily 4 P.M. spot price for heating oil (diesel fuel) at a specified location.

Suppose the current spot price for heating oil/jet fuel is $0.74 per gallon and assume a $0.04 swap spread. Then, each month for the duration of the swap, the airline's obligation is to pay the swap dealer $0.76 per gallon of jet fuel and the refiner has the obligation to sell heating oil to the swap dealer for $0.72 per gallon. In turn, the swap dealer has the obligation to pay the airline the average floating payment and refiner has the obligation to pay the swap dealer the average floating payment. The net effect of these obligations is that the airline pays a fixed price of $0.76 per gallon for jet fuel over the next three years and the refiner receives $0.72 per gallon for heating oil over the next three years.

The swap dealer faces a risk here in that the notional amounts are not equal. The swap dealer would either have to find an appropriate counterparty for the extra 50,000 gallons or hedge this risk in the heating oil futures market.

12.4 Using Equity Swaps

In an equity swap, the fixed-price payer makes periodic fixed interest payments on a notional principal. The floating-price payer makes periodic payments that are determined by the total return on a given stock index. By agreement, the total return could exclude dividends. The notional principal is not exchanged between the counterparties to the equity swap. Note that the equity index used is negotiable as well. Therefore, it could be a broad-based index like the S&P 500, the Nasdaq 100, or the Russell 2000. Or, it could be a foreign index like the Nikkei in Japan, the DAX in Germany, or the CAC in France. The equity index could even be a specific sector, like a bank index, a biotech index, or a pharmaceutical index, or a specific diversified portfolio of stocks.

For example, suppose a manager of an equity portfolio want to reduce his downside risk exposure from the collection of semiconductor stocks in his portfolio. This fund manager has a long-term bullish outlook on the semiconductor sector but feels that the sector could suffer some losses over the next year. Further, suppose the returns on the $25 million worth of semiconductor stocks in the fund manager's portfolio have a correlation coefficient of about 0.95 with the semiconductor index constructed by the Philadelphia Stock Exchange.[6]

Instead of selling the stocks in the portfolio (and paying the capital gains tax), and instead of using options on the Semiconductor Index (SOX), the fund manager decides to enter into an equity swap with a 12-month tenor. Under the terms of the swap agreement, the fund manager agrees to receive a fixed payment of 8% and pay the swap dealer the capital gain return on the semiconductor index (i.e., the dividends paid by any component stocks of the index are excluded). Payments are made quarterly and the notional principal is set at $25 million.

Using an equity swap, the fund manager has transformed the semiconductor portion of his portfolio into a quasi-riskless asset that yields 8% for one year. Note that the transformation will not be entirely riskless unless the returns on the actual equity portfolio equal the SOX index returns times the notional principal.

Further note that because the return on the semiconductor index could be positive or negative, the floating-rate payer (i.e., the fund manager) could make a payment or receive a payment from the swap dealer. For example, suppose the quarterly returns on the semiconductor index are 5.75, 16.75, −3.00, and 14.50%. Further, assume the actual portfolio returns equal the semiconductor index returns and that the beginning dollar value of the actual portfolio equals the notional principal. Then,

Quarter	(Payment to) Receipt from Swap Dealer	Receipt from Swap Dealer	Net Payment
First	($1,437,500)	$500,000	($937,500)
Second	($4,187,500)	$500,000	($3,687,500)
Third	$750,000	$500,000	$1,250,000
Fourth	($3,625,000)	$500,000	($3,125,000)
	($8,500,000)	$2,000,000	($6,500,000)

Ignoring the time value of money effects, if the gains on the actual portfolio holdings during the year equal $8,500,000, the portfolio manager has effectively turned his semiconductor stocks into a riskless asset yielding 8%. Of course, if the gains on the actual portfolio are not equal to the payments to the swap dealer, the portfolio manager's returns will not equal 8%.

12.5 Using Index Swaps

A pension plan owns a great deal of real estate. It wants to reduce its exposure to real estate because it forecasts rising interest rates and declining rental income, which will depress real estate values. It feels that having 20% of its portfolio in real estate represents too much exposure to this asset class. On the other hand, it does not want to sell the real estate now, because it would generate a huge amount of capital gains, and it does not want to pay capital gains taxes now; it would rather defer those taxes for a few years. Also, real estate is an illiquid asset class that often cannot be sold quickly without price concessions.

This pension plan can use an index swap to manage its risk. It can enter into a swap in which it pays the swap dealer the rate of return on a real estate value index[7] and receives LIBOR.

If the pension plan expects to sell off some of its properties over time, as conditions permit, it can estimate this timetable and let the notional principal of the index swap decline as the index swap approaches maturity. This is an example of an amortizing swap, but note that the pension plan faces the risk that its real estate will be sold at a rate that is faster or slower than anticipated.

As another example, consider a mutual fund based in Great Britain that would like to invest in the U.S. stock market. However, if it directly buys U.S. equities, it will have to pay a 15% withholding tax on all dividends it receives (U.S. investors in British stocks have to pay corresponding 15% tax). The British mutual fund can use an equity index swap to circumvent this tax. It can agree to swap payments and receive the returns on a U.S. stock index. It might agree to make payments based on returns on a British stock index if it would otherwise fund its U.S. investment by selling British stocks. It might agree to make payments based on GBP LIBOR if it otherwise funds its U.S. investments by either selling British bonds or borrowing GBP. It might instead agree to make payments based on U.S. dollar LIBOR if it thought that U.S. interest rates were going to fall, if it would otherwise have funded its U.S. equity investment by borrowing in the United States, or if it had U.S. debt securities it was planning on selling.

12.6 USING DIFF SWAPS

Diff swaps allow a firm to eliminate currency risk exposure when engaging in a currency swap. Recall from Chapter 11 that a differential swap, commonly called a diff swap, is a floating–floating currency swap. However in a diff swap, both floating rates are applied to the notional principal of just one currency, and the swapped payments are only in that currency. Because only one currency is involved, the notional principal is not exchanged, and the exchanged cash flows are netted, so only the difference check is paid from one party to the other.

For example, a firm may wish to make quarterly payments based on three-month Euribor, and receive three-month U.S. dollar LIBOR. Because three-month Euribor rates are less than three-month U.S. dollar LIBOR rates, the firm will have to pay a spread. Suppose this spread is 200 basis points. Further, suppose the firm wants to have both the payments and the receipts denominated in dollars. The swap's tenor is two years, payments are quarterly, and the day count method is 30/360. Assume that rates are determined as in an in-arrears swap. That is, the time t floating interest rates determine the time t cash flows. The notional principal of the swap is $150 million.

At each settlement date, both Euribor and U.S. dollar LIBOR rates are observed, and these two rates are netted to determine the net payment or receipt, in dollars. Suppose we observe the following sequence of interest rates over the next two years:

Quarter	**Euribor Rate**	**Firm Pays**	**LIBOR Rate**	**Firm Receives**	**Net Flow**
1	4.77%	$2,538,750	7.16%	$2,685,000	$146,250
2	4.95%	$2,606,250	7.26%	$2,722,500	$116,250
3	5.11%	$2,666,250	7.32%	$2,745,000	$78,750
4	5.30%	$2,737,500	7.38%	$2,767,500	$30,000
5	5.36%	$2,760,000	7.35%	$2,756,250	($3,750)
6	5.46%	$2,797,500	7.36%	$2,760,000	($37,500)
7	5.54%	$2,827,500	7.37%	$2,763,750	($63,750)
8	5.66%	$2,872,500	7.43%	$2,786,250	($86,250)

At the first payment date, three-months from now, the three-month U.S. LIBOR is 7.16% and the three-month Euribor is 4.77%. The firm is required to pay

$$\frac{0.0477 + 0.0200}{4} \times \$150,000,000 = \$2,538,750$$

However, the firm will receive

$$\frac{0.0716}{4} \times \$150,000,000 = \$2,685,000$$

Thus, the firm will receive the first difference check, for $146,250.

12.7 Summary

In this chapter, we have presented various examples of how firms can use swaps to manage risk. It is important to remember that swaps are negotiable. That is, the terms of a swap can be tailored to the needs and desires of the counterparties.

A firm can use interest rate swaps to lower borrowing costs, protect itself from an increase in future interest rates, or protect itself from a decrease in future interest rates. Floating-rate assets (liabilities) can be transformed into fixed-rate assets (liabilities). A firm can use currency swaps to decrease borrowing costs in a foreign country. In addition, a firm can use currency swaps to protect itself when a change in the price of a currency will decrease the (home-currency-denominated) value of a revenue stream, increase a cost, decrease the value of an asset, or increase in the value of a liability.

In this chapter, we also provided examples of using commodity swaps, equity swaps, index swaps, and diff swaps.

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Notes

[1]The growth of the swap market suggests that there is a tremendous economic benefit of these derivatives. Smithson (1999) summarizes several explanations for the quality spread. These include comparative advantage, risk shifting, differential cash flow packages, informational asymmetries, and tax and regulatory arbitrage.

[2]It is assumed that the BBB Company can lock in this floating rate (LIBOR + 75 basis points) for the entire tenor of the swap and its floating-rate bond issue.

[3]Recall from Chapter 11 that it is customary for LIBOR to be bought or sold flat in a swap. The fixed rate is the price paid or received for LIBOR.

[4]Smithson, Smith, and Wilford (1995) cite a 1993 survey by *Treasury* magazine that reports that 64% of the responding firms use derivatives to reduce funding costs.

[5]Gap analysis can be treacherous when the issuer of a security has the option to call the security for early prepayment. The question of when the asset or liability will be repriced becomes a random variable that depends on the level of interest rates or other factors.

[6]The Semiconductor Index is a price-weighted index that was created by the Philadelphia Stock Exchange. Options on the SOX Index began trading on September 4, 1994. As of May 9, 2001, the 16 stocks in the index were Altera

Corp. (ALTR), Applied Materials, Inc. (AMAT), Advanced Micro Devices (AMD), Intel Corporation (INTC), KLA-Tencor Corp. (KLAC), Linear Technology Group (LLTC), Lattice Semiconductor (LSCC), LSI Logic Corp. (LSI), Motorola, Inc. (MOT), Micron Technology, Inc. (MU), National Semiconductor (NSM), Novellus Systems, Inc. (NVLS), Rambus, Inc. (RMBS), Teradyne, Inc. (TER), Texas Instruments, Inc. (TXN), and Xilinx, Inc. (XLNX). For more information on the SOX, see the website of the Philadelphia Stock Exchange (www.phlx.com).

[7]There are several real estate value indexes. See, for example, the National Council of Real Estate Investment Fiduciaries website (www.ncreif.com).

PROBLEMS

12.1 Popsico has a great deal of floating-rate debt outstanding. Currently, the yield curve is upward sloping. Popsico believes that for the next year interest rates will remain low and the yield curve will remain upward sloping. Beyond a year, Popsico fears that short-term interest rates will begin to rise sharply. What can Popsico do both to take advantage of the current low short-term rates and to hedge against the risk that short-term rates will begin to rise in a year or two?

12.2 Electronic Business Machines (EBM) needs to borrow $20 million immediately. It can borrow for three years at a fixed rate of 7.5% or at a floating rate of LIBOR + 40 basis points. Plain vanilla fixed-for-floating three-year swaps are priced at 7.3% fixed, in exchange for floating LIBOR. If EBM believes that interest rates are about to rise sharply, what should it do? If EBM believes that interest rates are about to decline sharply, what should it do?

12.3 Atlantis-Morris Seats is a chair manufacturer that is too small to borrow in the long-term fixed-rate debt market. All its borrowed funds have been obtained by borrowing from banks, which change the rates they charge depending on market conditions. Explain how this firm can use the swaps market to obtain fixed-rate funds.

12.4 The Goo-Goo Doll Manufacturing Company is a major toy maker. Eight years ago, it borrowed $30 million for 10 years at a fixed rate of 10%. Today, dealers are quoting fixed-for-floating interest rate swaps as follows:

Swap Tenor	Yield on Treasury Securities	Swap Spreads (Basis Points) Bid	Ask
1 year	4.8%	20	24
2 year	5.3%	25	29
3 year	5.7%	27	32
⋮	⋮	⋮	⋮
8 years	6.4%	31	36
10 years	6.5%	33	38

If the Goo-Goo Doll Company prefers to have floating-rate debt for the next two years, how can it use the swap market to accomplish this? If the firm believes that interest rates are going to rise about 200 basis points over the next year, should it use a swap to convert its fixed-rate debt to synthetic floating-rate debt?

12.5 Why do swap dealers exist? Why don firms just swap with themselves?

12.6 A firm enters into a typical plain vanilla, fixed–fixed currency swap. It is paying yen at a fixed rate and receiving dollars at a fixed rate. This firm likely entered into this swap because

a. It had a comparative advantage borrowing in yen, or faced the risk that a rising ¥/$ exchange rate would hurt profits.

b. It had a comparative advantage borrowing in yen, or faced the risk that a falling ¥/$ exchange rate would hurt profits.

c. It had a comparative advantage borrowing in dollars, or faced the risk that a rising ¥/$ exchange rate would hurt profits.

d. It had a comparative advantage borrowing in dollars, or faced the risk that a falling ¥/$ exchange rate would hurt profits.

12.7 Suppose that you are a U.S. citizen concerned about personal U.S. dollar-denominated wealth and you own a large portfolio of Canadian utility stocks that are paying quarterly dividends, denominated in Canadian dollars. Do you face the risk that the Canadian dollar will get stronger or weaker? Why? To hedge, you have decided to engage in a swap with no initial and no terminating exchange of principal. Will you be the party that is buying U.S. dollars or selling U.S. dollars?

CHAPTER 13

Pricing and Valuing Swaps

Determining the theoretical price of a swap is quite simple. In Chapter 11, you learned that a swap is equivalent to an asset and a liability and that an at-market swap has zero value (ignoring the impact of the bid–ask spread) when it is originated. This means that to determine the fixed price for a swap, a firm should find the present value of the swap's payments and the swap's receipts. The price that equates the two present values results in a zero net value for the two parties. Then the swap dealer, to make a profit in her service as a market maker, can adjust the price to quote a bid–ask spread. Thus, valuing swap cash flows involves no more work than valuing two sets of cash flows.[1]

After its origination, the swap can be valued by comparing the new present value of the remaining cash payments to the new present value of the remaining cash receipts. *Current* interest rates and exchange rates must be used to compute these two present values. The remaining *fixed* cash flows are the same as they were when the swap was originated, although they will be discounted at new, current rates. The remaining expected *floating* cash flows will likely be different, and they too will be discounted at new current rates. Remember that a swap is a **zero sum game**, so that if the swap has a positive value for one party, it must have the same negative value for the counterparty.

Another way to value swaps after origination is to compare the price of the old swap with the current price for a swap with a tenor equal to the remaining time to maturity of the old swap. Then, one compares the present value of the old swap's fixed cash flows with the present value of the fixed cash flows for a new swap. This shortcut is valid because the same LIBOR serves as the floating rate in both cases.

If valuation is properly done, the value of the swap will be the same regardless of which approach is used. The differences between the two approaches will become apparent when an example is worked out, later in the chapter.

This chapter begins with an example of the method that prices and values plain vanilla fixed–floating interest rate swaps.

13.1 PRICING AND VALUING PLAIN VANILLA FIXED–FLOATING INTEREST RATE SWAPS

In a fixed–floating interest rate swap, one party agrees to make a series of fixed payments and receive a series of floating, or variable, payments. Thus, the goal of swap pricing is to determine the fixed interest rate that makes the present value of the fixed payments equal to the present value of the floating receipts. At its origination, the theoretically correct swap price will create a swap with zero value (unless it is an off-market swap).

Finding the present value of the fixed payments is a straightforward problem of finding the present value of an annuity because the fixed payments are known as soon as the swap price is set. However, only the first floating cash flow is known because, in a typical fixed–floating interest rate swap, the floating rate at origination establishes the first cash flow. Because the remaining floating cash flows are unknown, the floating cash flows must either be estimated or, more accurately, set equal to the cash flows the swap dealer will face if the swap is hedged. For example, a swap dealer who will *receive* a series of floating cash flow may be able to use forward rate agreements (FRAs) to *pay* the same floating cash flows. In this case, the swap dealer has hedged the floating cash flows. If interest rates decline, the swap dealer will receive less from the floating-rate swap payments but will also have to pay out less on the FRAs. Because he is hedged, he can use the FRA prices as the floating interest rates.

13.1.1 An Example of Fixed–Floating Interest Rate Swap Pricing

Recall that the spot yield curve is used to determine theoretical futures prices and FRA prices. Thus, it should be apparent that swap dealers can use FRAs, futures prices, and spot zero coupon interest rates to determine swap prices. An example illustrates the steps needed to determine swap prices:

Define

$r(0, t)$ = spot interest rate for a zero coupon bond maturing at time t

$fr(t1, t2)$ = forward interest rate from time $t1$ until time $t2$

Suppose the spot term structure is $r(0,1)=5\%$, $r(0,2)=6\%$, $r(0,3)=7.5\%$. From these spot rates on pure discount debt instruments, the one year forward rate, one-year hence, can be computed to be $fr(1,2)=7.01\%$:

$$(1.06)^2 = (1.05)[1 + fr(1,2)]$$

$$fr(1,2) = 0.070095$$

The one-year forward rate, two years hence, is $fr(2,3) = 10.56\%$:

$$(1.075)^3 = (1.06)^2[1 + fr(2,3)]$$

$$fr(2,3) = 0.10564$$

Now consider a three-year swap. The floating rate is one-year LIBOR. Settlement payments are made yearly. What is the fair fixed rate of the swap (i.e., the swap price)?

For conceptual ease only, it is sufficient to assume that the pure expectations theory of the term structure of interest rates is correct. Thus, forward rates equal expected future spot rates, and forward rates can be used to compute expected floating cash flows.[2]

Arbitrarily, suppose the swap's notional principal is $100. Thus, the expected floating cash flows are as follows:

$$CF_1 = 0.05 \times 100 = 5$$

$$CF_2 = 0.070095 \times 100 = 7.0095$$

$$CF_3 = 0.10564 \times 100 = 10.5640$$

Next, value these floating cash flows at the appropriate discount rates, the spot zero coupon interest rates:

$$\frac{5}{1.05} + \frac{7.0095}{1.06^2} + \frac{10.5640}{1.075^3} = 19.50394$$

Since the value of an at-market swap at its origination is zero, the value of the floating payments must equal the value of the fixed payments. Therefore, the fixed-rate payments must satisfy the following:

$$19.50394 = \frac{A}{1.05} + \frac{A}{1.06^2} + \frac{A}{1.075^3} = A\left(\frac{1}{1.05} + \frac{1}{1.06^2} + \frac{1}{1.075^3}\right)$$

$$2.647338A = 19.50394$$

$$A = 7.36738$$

This means that if a swap dealer agrees to make fixed payments of 7.3674% of the notional principal amount, and receive floating payments, the values of the two cash flow streams are equal.[3]

To make a profit, the swap market maker might quote prices of 7.34% to a fixed-rate receiver and 7.39% to a fixed-rate payer. Recall from Chapter 11 that interest rate swaps are typically quoted as a spread over the yield to maturity of Treasury securities. Thus, if the yield on the most recently issued three-year Treasury note is 7.05%, the quoted swap spreads might be 29/34. The 29 is the bid quote, and this means that a firm that wishes to be the receive-fixed counterparty will receive 29 basis points over 7.05%, which is 7.34%. If the firm wishes to be the pay-fixed counterparty, it will have to pay 34 basis points over 7.05%, which is 7.39%.

The steps in pricing an at-market swap can be summarized as follows:

1. Find the present value of the floating-rate cash flows. Use the forward rates to determine the floating amounts to be exchanged and the spot rates on zero-coupon bonds as the appropriate discount rates.
2. Find the fixed rate that makes the present value of the fixed payments equal to the figure computed in step 1. The fixed rate determines the fixed amounts to be exchanged, and the spot rates are used on zero-coupon bonds as the appropriate discount rates.

13.1.2 A Shortcut to Pricing At-Market Fixed–Floating Interest Rate Swaps at Origination

It is important to understand the pricing model presented in Section 13.1.1. Later, we will see that it is needed to value off-market swaps and to value a swap after its origination date. However, there is a shortcut to pricing at-market fixed for floating interest rate swaps.

The key to understanding why this shortcut works is as follows. The value of a floating rate bond is approximately equal to its face value, because the next cash flow is always repriced to reflect the level of current interest rates.[4]

This last point can be more easily understood by using a simple example. Consider a two-year floating-rate bond that will pay annual interest. The dollar floating coupon amount paid one year hence is based on today's one-year rate. The amount paid two years hence is calculated based on the one-year rate one year hence, which is currently unknown. For conceptual ease, assume that the one-year spot rate that will actually exist one year hence is expected to equal today's one-year forward rate, for delivery one year hence. In other words, the pure expectations theory of the term structure is appropriate. Let $r(0,1)=5\%$, and $fr(1,2)=8\%$. The face value of the bond is \$100. The value of this bond is

$$\frac{5}{1.05}+\frac{108}{(1.05)(1.08)}=100$$

If interest rates change so that, say, $r(0,1)=6\%$ and $fr(1,2)=7.5\%$, the value of the bond is still \$100:

$$\frac{6}{1.06}+\frac{107.5}{(1.06)(1.075)}=100$$

Thus, we have shown that the value of a floating rate bond will always be very close to its par value. Because of this, the swap pricing problem is reduced to finding the fixed payment that makes the value of a fixed-coupon bond equal to its par value. The discount rates are the spot rates on zero-coupon bonds.

Returning to the example in Section 13.1.1, the problem is to solve for the value of the unknown, A, that makes the value of the fixed cash flows on an ordinary fixed coupon bond equal to 100:

$$100=\frac{A}{1.05}+\frac{A}{1.06^2}+\frac{A}{1.075^3}+\frac{100}{1.075^3}$$

$$100=A\left(\frac{1}{1.05}+\frac{1}{1.06^2}+\frac{1}{1.075^3}\right)+80.4961$$

$$19.5039=2.647338A$$

$$A=7.36736$$

The example concludes that the fixed rate that should be quoted for the swap is 7.3674%. Except for some minor rounding errors, this is exactly what we computed before.

The FinancialCAD function aaParSwap can be used to price a plain vanilla interest rate swap.[5] Figure 13.1 illustrates how this is done.

13.1.3 Valuing an Interest Rate Swap After Origination

Interest rates will almost certainly change after the swap's initiation. As interest rates change, the swap value changes. This section explains how to value a swap after swap initiation. The valuation method is the same as the one used in Section 13.1.1. That is, we can either find the present value of the swap's remaining payments and receipts or find the present value of the fixed payments on a newly originated swap, and compare them to the present value of the fixed payments on the old swap. Both methods yield the same result.

Valuing swaps after they have been originated is important for many related reasons. First, firms wish to know their current financial position. Even if a swap is an off-balance sheet transaction, firms will want to know the true value of all their assets and liabilities, even those that are off-balance sheet items. Second, firms wish to know the default risk they face. If a swap has become

AaParSwap		
Settlement date	24-Sep-2000	
Terminating date	24-Sep-2003	
effective date	24-Sep-2000	
date of first coupon after dated date		
date of last coupon prior to maturity date		
cash flow frequency	1	Annual
accrual method for coupons	1	actual/365 (fixed)
accrual method for accrued interest	1	actual/365 (fixed)
holiday list	t_26	
business day convention (see Glossary)	1	no date adjustment
interpolation method	1	Linear
discount factor curve	t_43_1	

t_26
holiday list

holiday date
1-Jan-1995

t_43_1
discount factor curve

grid date	Discount factor	
24-Sep-2000	1	
24-Sep-2001	0.952380952	=1/1.05
24-Sep-2002	0.88999644	=1/(1.06)^2
24-Sep-2003	0.80496057	=1/(1.075)^3

par swap rate	0.073673794

Figure 13.1 The FinancialCAD function aaParSwap solves for the fixed rate of a plain vanilla, fixed-for-floating, interest rate swap. The key is to enter the proper discount factors.

a net asset for them, they will become more concerned about the ability of the counterparty to make its payments. Third, many swaps are marked to market. This requires valuing swaps after they have been originated.

As in the last example, the spot term structure is $r(0,1)=5\%$, $r(0,2)=6\%$, $r(0,3)=7.5\%$. Now, the problem is extended by letting the four-year spot interest rate, $r(0,4)$, equal 8%. In the last example, we computed $fr(1,2)=7.0095\%$ and $fr(2,3)=10.5640\%$. The one-year forward rate, for delivery three years hence, is $fr(3,4)=9.5140\%$:

$$1.08^4 = 1.075^3[1+fr(3,4)]$$

$$fr(3,4) = 0.095140$$

The swap's notional principal is \$30 million and payments are annual. The price of a four-year plain vanilla fixed–floating interest rate swap is 7.8339%, computed by using the shortcut method in Section 13.1.2 as follows:

$$100 = \frac{A}{1.05} + \frac{A}{1.06^2} + \frac{A}{1.075^3} + \frac{A}{1.08^4} + \frac{100}{1.08^4}$$

$$100 = A\left(\frac{1}{1.05} + \frac{1}{1.06^2} + \frac{1}{1.075^3} + \frac{1}{1.08^4}\right) + 73.503$$

$$26.497 = 3.38237A$$

$$A = 7.83386$$

The swap dealer quotes a price 7.83% in exchange for one-year LIBOR. There is no bid–asked spread. The dollar amount of each fixed payment is (0.0783)($30,000,000) = $2,349,000. Now suppose that one year after origination, the term structure has changed, and $r(0,1) = 4.5\%$, $r(0,2) = 5\%$, and $r(0,3) = 5.5\%$. What is the value of the swap for the pay-fixed counterparty? This is answered in two ways.

13.1.3.1 Valuing an Interest Rate Swap After Origination: Method 1

In the first method, find the present value of the remaining fixed cash payments and the present value of the remaining expected floating receipts. Both these are discounted at current spot interest rates to find the present value. The difference between these two present values is the value of the swap.

The annual fixed payments are: (0.0783) ($30,000,000) = $2,349,000. The present value is $6,378,900:

$$\frac{2,349,000}{1.045} + \frac{2,349,000}{1.05^2} + \frac{2,349,000}{1.055^3} = \$6,378,900$$

The present value of the expected remaining floating payments is computed as follows. First, it is necessary to find the forward rates that will exist one year after the swap's origination:

$$1.05^2 = 1.045[1 + r(1,2)]$$

$$fr(1,2) = 0.055024$$

$$1.055^3 = 1.05^2[1 + r(2,3)]$$

$$fr(2,3) = 0.065072$$

Therefore, the present value of the expected floating receipts is[6]:

$$\frac{(0.045)(\$30,000,000)}{1.045} + \frac{(0.055024)(\$30,000,00)}{1.05^2} + \frac{(0.065072)(\$30,000,000)}{1.055^3} = \$4,451,604$$

Because the present value of the fixed payments is $6,378,900, and the present value of the floating receipts is only $4,451,604, the swap, which now has a negative value of $1,927,296 has become a liability for the pay-fixed party. Of course, the swap is a positive value asset for the receive-fixed counterparty.

13.1.3.2 Valuing an Interest Rate Swap After Origination: Method 2

In the second method, we find what swap dealers are now quoting (one year after the swap's origination) as a fixed rate to be swapped for one-year LIBOR. This new swap should have a tenor of only three years. The difference between the original swap's fixed payments and a new swap's fixed payments equals the original swap's value.

Using the shortcut method to value an at-market swap, we find that the new price is 5.464258%:

$$100 = \frac{A}{1.045} + \frac{A}{1.05^2} + \frac{A}{1.055^3} + \frac{100}{1.055^3}$$

$$100 = A\left(\frac{1}{1.045} + \frac{1}{1.05^2} + \frac{1}{1.055^3}\right) + 85.1614$$

$$14.8386 = 2.7156A$$

$$A = 5.464258$$

This means that given the notional principal of \$30 million, the fixed payments on a new swap would be (0.05464258)(\$30,000,000) = \$1,639,277. The difference between the old swap's fixed payments and the new swap's fixed payments is \$2,349,000 − \$1,639,277 = \$709,723 per year. The present value of the difference in fixed cash flows, given the new discount rates is \$1,927,310:

$$\frac{\$709,723}{1.045} + \frac{\$709,723}{1.05^2} + \frac{\$709,723}{1.055^3} = \$1,927,310$$

Except for some rounding errors, this is the same number we computed by using method 1 (Section 13.1.3.1). Thus, if this swap were marked to market after one year, the pay-fixed party would have to pay \$1,927,310 to the receive-fixed counterparty. If the pay-fixed party approached a swap dealer and asked to get out of the swap contract early, the swap dealer would agree for a payment of \$1,927,310.

Note that if spot interest rates were $r(0,1)=4.5\%$, $r(0,2)=5.0\%$, and $r(0,3)=5.5\%$, and if a firm approached a swap dealer requesting to be the pay-fixed party in an off-market swap with a price of 7.83%, the swap dealer would pay the firm \$1,927,310. This up-front payment would be compensation for the firm because it has asked to pay 7.83% fixed, when the market price is only 5.464258%.

The FinancialCAD function aaSwpi provides the same answer for valuing an existing swap after time has passed and the term structure of interest rates has changed, as in the example we just covered. Figure 13.2 shows the solution.

13.1.4 Using FRAs and Futures to Price Swaps

Theoretical forward interest rates are derived from spot interest rates for pure discount bonds. Futures interest rates are theoretically equal to forward interest rates, except for the impact of marking futures contracts to market daily, which could have a small impact on theoretical futures prices. Thus, the prices of FRAs, as well as futures interest rates and spot interest rates, can all be used to price swaps.

AaSwpi		
value (settlement) date	24-Sep-2001	
effective date	24-Sep-2001	
terminating date	24-Sep-2004	
Structure	1	pay floating and receive fixed
interpolation method	1	linear
exchange of principal	1	no exchange
margin above or below a floating rate	0	
FX spot - pay / receive	1	
Coupon	0.0783	
Statistic	1	fair value
current fixing of rate as of last reset date	0.045	
business day convention (see Glossary) - pay leg	1	no date adjustment
direction for date generation - pay leg	1	forward from effective date
cash flow frequency - pay leg	1	annual
odd date list - pay leg	t_32	
accrual method - pay leg	4	30/360
holiday list - pay leg	t_26	
notional principal - pay leg	30000000	
discount factor curve - pay leg	t_43_1	
business day convention (see Glossary) - receive leg	1	no date adjustment
direction for date generation - receive leg	1	forward from effective date
cash flow frequency - receive leg	1	annual
odd date list - receive leg	t_32	
accrual method - receive leg	4	30/360
holiday list - receive leg	t_26	
notional principal - receive leg	30000000	
discount factor curve - receive leg	t_43_1	

t_32
odd date list
non-cycle dates on which a cash flow occurs

t_26
holiday list

holiday date
1-Jan-1995

t_43_1
discount factor curve

grid date	discount factor	
24-Sep-2001	1	
24-Sep-2002	0.956937799	=1/1/045
24-Sep-2003	0.907029478	=1/(1.05)^2
24-Sep-2004	0.851613664	=1/(1.055)^3

t_32
odd date list
non-cycle dates on which a cash flow occurs

t_26
holiday list

holiday date
1-Jan-1995

t_43_1
discount factor curve

grid date	discount factor	
24-Sep-2001	1	
24-Sep-2002	0.956937799	=1/1/045
24-Sep-2003	0.907029478	=1/(1.05)^2
24-Sep-2004	0.851613664	=1/(1.055)^3

fair value	1927309.558

Figure 13.2 The FinancialCAD function, aaSwpi, solves for the value of a swap after it has been originated, and interest rates have changed.

In practice, swap dealers will hedge their floating cash flows. They will hedge them by using either FRAs or futures, depending on factors such as relative prices and liquidity. Another relevant factor is that Eurodollar futures have specific delivery dates, and they will likely not coincide with the dates on which the swap cash flows will be exchanged. Swap dealers who are planning to use these contracts to hedge a swap's floating cash payment or receipt will price their swaps from FRA prices or futures prices.

EXAMPLE 13.1 Pricing A Forward Swap Using a Strip of Eurodollar Futures Contracts A forward swap is one that is executed today, but the cash flows begin at a future date (beyond what you expect in a plain vanilla swap). Forward swaps are also often called **forward start swaps**. They are equivalent to two partially offsetting swaps, both of which begin today, but one is longer than the other. Thus, a two-year payer swap and a five-year receiver swap together create a three-year receiver swap that begins two years hence.

A swap is equivalent to a strip of futures contracts. Often, the price of a swap is obtained by using the prices of futures contracts. Here, we show how Eurodollar futures prices can be used to determine the theoretical price of a forward, fixed-for-floating interest rate swap.

For our example, consider the following. On July 28, 1999, suppose a firm has committed to borrow $50 million with a floating interest rate, for two years, beginning in September. The firm now wishes it had borrowed at a fixed rate. The firm approaches a swap dealer to get a price for a fixed-for-floating swap, expressing the desire to be the pay-fixed party.

Under the current conditions of the loan, the interest rate is set quarterly to the prevailing spot three-month LIBOR rate. In addition, the date on which the floating interest rate is set coincides with the settlement dates of Eurodollar futures contracts. Assume for simplicity that the interest expense is calculated by using a 30/360 day count method. Also assume that the first interest expense payment will be made three months after the loan begins (i.e., in December 1999).

Based on the Eurodollar futures settlement prices from Table 10.2, the swap dealer can generate the following information:

Contract in Strip?	Month/ Strike	Euro dollars Futures Settlement	Yield Settle	Unannualized Forward Rate	Discount Factor	Inverse of Discount Factor
No	Aug 1999	94.625	5.375	—	—	—
Yes	Sep 1999	94.555	5.445	0.013613	1.013613	0.986570
No	Oct 1999	94.285	5.715	—	—	—
No	Nov 1999	94.255	5.745	—	—	—
Yes	Dec 1999	94.190	5.810	0.014525	1.028335	0.972446

No	Jan 2000	94.315	5.685	—	—	—
Yes	Mar 2000	94.185	5.815	0.014538	1.043285	0.958511
Yes	June 2000	93.950	6.050	0.015125	1.059064	0.944230
Yes	Sept 2000	93.765	6.235	0.015588	1.075572	0.929737
Yes	Dec 2000	93.535	6.465	0.016163	1.092956	0.914950
Yes	Mar 2001	93.550	6.450	0.016125	1.110580	0.900430
Yes	June 2001	93.490	6.510	0.016275	1.128655	0.886010

Note that the quarterly payments of the swap determine which futures contracts should be in the strip that is equivalent to the swap. The September Eurodollar futures contract is linked to the three-month interest rate that will exist beginning in September, and this is the same interest rate that determines the first cash flow of the swap, in December.

Each unannualized forward rate is calculated as follows:

$$\text{unannualized forward rate}_i = \left(\frac{\text{yield settle}_i}{100}\right)\left(\frac{90}{360}\right)$$

and the discount factors are calculated by compounding the unannualized forward rates. For example, the entry for the March 2000 discount factor is given by

March 2000 discount factor = 1.013613 × 1.014525 × 1.014538 = 1.043285 and the inverse discount factor column is simply

$$\text{inverse}_i = \frac{1}{\text{discount factor}_i}$$

For example, the inverse of the March 2000 discount factor is 1/1.043285 = 0.958511. In addition, the swap dealer can calculate the floating payments as follows:

$$\text{floating payment}_i = \text{unannualized forward rate}_i \times \$50{,}000{,}000$$

For example, the last floating payment of \$813,750 is computed using 0.016275 × \$50,000,000 = \$813,750. The floating payments are then discounted using the discount factors. These discounted floating payments are the present value (as of September 1999) of the floating payments:

$$\text{discounted floating payment}_i = \frac{\text{floating payment}_i}{\text{discount factor}_i}$$

With these formulas, the swap dealer can generate the following additional information:

Borrowing Period Starting	Floating Payment	Discount Factor	Discounted Floating Payment
Sept 1999	\$680,625	1.013613	\$671,484 (=\$680,625/1.013613)
Dec 1999	\$726,250	1.028335	\$706,239
Mar 2000	\$726,875	1.043285	\$696,718
June 2000	\$756,250	1.059064	\$714,074

Sept 2000	\$779,375	1.075572	\$724,614
Dec 2000	\$808,125	1.092956	\$739,394
Mar 2001	\$806,250	1.110580	\$725,972
June 2001	\$813,750	1.128655	\$720,991 (=\$813,750/1.128655)
Total			\$5,699,486

The swap dealer can now calculate the fixed payment each 90 days that would have the same present value as the floating payments. That is, solve the following equation for F:

$$F\left(\sum_i \frac{1}{\text{discount factor}_i}\right) = F\left(\sum_i \text{inverse}_i\right) = \$5,699,486$$

$$F(7.492884) = \$5,699,486$$

$$F = \$760,653$$

The fixed rate is ther given by:

$$\text{fixed rate} = \left(\frac{\text{fixed payment}}{\text{notional value}}\right)\left(\frac{360}{90}\right)$$

$$\text{fixed rate} = \left(\frac{\$760,653}{\$50,000,000}\right)4$$

$$\text{fixed rate} = 6.08522\%$$

Note that the swap dealer could approximate an unannualized fixed rate with an IRR calculation. In this calculation, the notional value is the cash outlay beginning at the loan's initiation in September. Beginning in December, each quarterly floating payment and the notional value are subsequent cash inflows:

$$0 = -\$50,000,000 + \left(\sum_i \frac{\text{floating payment}_i}{(1+F_{\text{IRR}})^i}\right) + \frac{\$50,000,000}{(1+F_{\text{IRR}})^8}$$

$$F_{\text{IRR}} = 1.5214\%$$

$$F \approx 1.5214\% \times 4 \approx 6.0856\%$$

This IRR approach is a shortcut in the estimation of the theoretical swap price, using futures prices. The shortcut solution of 6.0856% is not much different from the more precise rate of 6.08522%, which we computed earlier. You are asked to use excel to compute the IRR solution in end-of-chapter Problem 13.14.

In this example, because the firm wants to pay a fixed rate, the swap dealer might quote a price of, say, 6.12%. Note that the firm itself has the opportunity to use Eurodollar futures contracts to"lock in" all the interest rate payments. If it does so, then each interest rate payment will be known, but the amount will be different at each date. If the firm does not want to go to the trouble of hedging the interest rate payments, and/or if the firm wants to have a known and constant interest rate payment, the firm will enter into the swap.

13.2 PRICING A CURRENCY SWAP

In a plain vanilla currency swap, the principal amounts, expressed in terms of two different currencies, are exchanged at origination at the prevailing spot exchange rate. Thus, this initial swap of principal amounts has no pricing or valuation consequences. If the current exchange rate is $1.61/£, then swapping $25,000,000 for £15,527,950 is currently a valueless transaction from the viewpoint of both parties to the swap.

Pricing a currency swap involves the same principles as pricing an interest rate swap. The present value of the payments must equal the present value of the receipts, so that an at-market swap has a value of zero at origination. If the principal amounts are again exchanged at the swap's termination (as they typically are), these amounts must be included in the valuation process. The present values can be expressed in either currency, and the link between the two present values is the exchange rate at origination. Either payment can be fixed or floating. A currency swap is equivalent to a firm selling, or issuing, a bond denominated in one currency and buying a bond in a different currency. When converted at the spot exchange rate, the values of the asset and the liability are equal.

13.2.1 An Example of Pricing a Fixed–Fixed Currency Swap

Consider the following swap:

Maturity	Three years
Principal	$25 million
Currency	deutschemarks
Fixed dollar rate	8%
Fixed deutschemark rate	To be determined
Payment dates	Every six months
Day count method	30/360

It is also necessary to know that the spot exchange rate is $0.70/DEM and that spot interest rates on zero-coupon debt instruments in each country are as follows:

Years to Maturity	United States	Germany
0.5	0.045	0.060
1.0	0.055	0.062
1.5	0.062	0.060
2.0	0.065	0.058
2.5	0.067	0.058
3.0	0.068	0.058

Now, the problem is to solve for the fixed rate, denominated in deutschemarks, that makes the value of the payments (in dollars) equal to the value of the receipts (in DEM), when valued at the current spot exchange rate of $0.70/DEM.

To proceed, note that the present value of the fixed dollar payments is $25,915,014. This is computed by discounting the six semiannual payments of $1 million each (at the spot U.S. interest rates) and also by discounting the principal value of $25 million to be swapped three years hence (at the termination of the swap). The semiannual payment is $1 million because the fixed U.S. dollar swap rate is 8%, payments are made semiannually, and the day count method is 30/360. Therefore, the semiannual interest rate is 4%, and 4% of $25 million is $1 million.

$$\frac{\$1,000,000}{1.045^{0.5}}+\frac{\$1,000,000}{1.055}+\frac{\$1,000,000}{1.062^{1.5}}+\frac{\$1,000,000}{1.065^{2}}+\frac{\$1,000,000}{1.067^{2.5}}$$
$$+\frac{\$26,000,000}{1.068^{3}}=\$25,915,014$$

Now, it is necessary to solve for the unknown, fixed German interest rate that determines the fixed periodic DEM cash flows that will be swapped. The principal amount, expressed in DEM, is based on the origination date exchange rate of $0.7/DEM:

$$\$25,000,000\,(\text{DEM}\,/\,\$0.7)=\text{DEM}35,714,286$$

Thus, DEM35,714,286 will be exchanged for $25 million both at origination and at termination. The exchange at origination has no impact. The exchange at termination must be included in the pricing problem. The present value of the fixed DEM cash flows, when converted back to dollars, is[7]:

$$\left[\begin{array}{l}\dfrac{x(\text{DEM}35,714,286)}{1.06^{0.5}}+\dfrac{x(\text{DEM}35,714,286)}{1.062}+\dfrac{x(\text{DEM}35,714,286)}{1.06^{1.5}}+\\ \dfrac{x(\text{DEM}35,714,286)}{1.058^{2}}+\dfrac{x(\text{DEM}35,714,286)}{1.058^{2.5}}+\dfrac{x(\text{DEM}35,714,286)}{1.058^{3}}+\\ \dfrac{\text{DEM}35,714,286}{1.058^{3}}\end{array}\right]\times\frac{\$0.7}{\text{DEM}}$$

Note that at time 3, *both* a payment of x(DEM35,714,286) *and* the principal amount (DEM35,714,286) are exchanged and discounted back three periods at the three-year rate of 5.8%. The term in the square brackets [ ] is the present value of the DEM cash flows, denominated in deutschemarks. The last term, $0.7/DEM converts this DEM value into dollars at the origination date spot exchange rate. The foregoing equation must equal the present value of the dollar cash flows. Setting them equal results in the solution for the unknown, fixed German interest rate that determines the fixed periodic DEM cash flows that will be swapped (i.e., x):

$$(\text{DEM}194,124,972x+\text{DEM}30,156,780)\$0.7/\text{DEM}=\$25,915,014$$

$$\text{DEM}35,887,480x=\text{DEM}4,805,268$$

$$x=0.035362$$

Thus, the fixed German interest rate is 3.5362%. This is the unannualized, half-year rate that equates the present value of the fixed dollar cash flows to the present value of the fixed DEM cash flows. The fixed DEM rate, expressed as an annual percentage, is then 7.0724%.

FinCAD can be used to find the same solution, but only in a roundabout way that utilizes Excel's goal seek tool.

13.2.2 Pricing a Fixed–Floating Currency Swap

To price a fixed-to-floating currency swap, find the fixed rate that makes the present value of the fixed cash flows equal to the present value of the floating cash flows. Because the floating cash flows are unknown, however, they are typically estimated by theoretical forward prices. Alternatively, and almost equivalently, they will be the forward prices or futures prices that swap dealers will have to pay or receive when they use forward foreign exchange contracts or futures contracts to hedge *their* risk exposures.

For example, consider a fixed–floating currency swap with the following structure:

Maturity	Three years
Principal	$25 million
Floating currency	Deutschemarks
Fixed dollar rate	To be determined
Payment dates	Every six months
Day count method	30/360

The spot exchange rate is $0.70/DEM, and spot interest rates on zero-coupon debt instruments in each country are as follows:

	Spot Interest Rates, r	
Years to Maturity, T	**U.S., r_{US}**	**Germany, r_G**
0.5	0.045	0.060
1.0	0.055	0.062
1.5	0.062	0.060
2.0	0.065	0.058
2.5	0.067	0.058
3.0	0.068	0.058

The goal is to solve for the fixed U.S. dollar interest rate that makes this at-market swap valueless from the viewpoint of both swap parties. First, we must solve for the theoretical forward exchange rates that exist, given the spot exchange rate and the term structures prevailing in each country. These theoretical forward prices are computed by using Equation (13.1) (which was originally introduced in Chapter 5):

$$F = S\left[\frac{1+r_{US}}{1+r_G}\right]^T \qquad (13.1)$$

where F and S are expressed as $/DEM, and $S = 0.70$.

Although they are not necessary to solve the problem, we can use the theoretical forward foreign exchange formula and the interest rates presented here to compute this set of forward exchange rates:

Years to Maturity	Forward Rate ($/DEM)
0.5	0.69503
1.0	0.69539
1.5	0.70198
2.0	0.70929
2.5	0.71498
3.0	0.72004

These values are the theoretical forward prices and futures prices. If a swap dealer is the pay-fixed party, he will receive floating DEM payments. He faces the risk that the dollar value of these future unknown DEM payments will decline. To hedge these risky DEM cash inflows, he can sell a strip of DEM forward contracts or sell a strip of DEM futures contracts. By selling DEM forward, the swap dealer locks in the selling price of the DEM cash flows.

To solve the swap pricing problem, forward German interest rates must be computed. These are found using Equation (13.2) (also introduced in Chapter 5):

$$[1+r(0,t2)]^{t2}=[1+r(0,t1)]^{t1}\times[1+fr(t1,t2)]^{t2-t1} \qquad (13.2)$$

Given the spot German interest rates, the annualized forward rates and the unannualized forward rates for semiannual compounding are:

$r(0.0, 0.5)$	0.06000	0.02956
$fr(0.5, 1.0)$	0.06400	0.03151
$fr(1.0, 1.5)$	0.05601	0.02762
$fr(1.5, 2.0)$	0.05202	0.02568
$fr(2.0, 2.5)$	0.05800	0.02859
$fr(2.5, 3.0)$	0.05800	0.02859

Each $fr(t1, t2)$ is a six-month forward interest rate for the period beginning at time $t1$ and ending at time $t2$. Today is represented as time 0. As an illustration of how the formula works, we will use Equation (13.2) to compute $fr(1.5, 2.0)$, the six-month forward rate that begins 18 months hence:

$$1.058^2 = 1.06^{1.5}[1+fr(1.5,2.0)]^{0.5}$$

$$1.119364 = 1.091337[1+fr(1.5,2.0)]^{0.5}$$

$$1.025682 = [1+fr(1.5,2.0)]^{0.5}$$

$$1.05202 = [1+fr(1.5,2.0)]$$

$$0.05202 = fr(1.5,2.0)$$

To obtain the semiannual forward rate from the annualized forward rate, use

$$\begin{aligned} fr_{\text{semi}}(t1,t2) &= [(1+fr(t1,\ t2))^{1/2}-1] \\ &= [(1+0.05202)^{1/2}-1] \\ &= 0.02568 \end{aligned}$$

If these semiannual forward German interest rates represent expected future six-month German spot interest rates, then they can be used to compute the estimated floating DEM swap cash flows.[8] Then, the present value, in DEM, of the expected floating cash flows is:

$$\begin{aligned} &\frac{(0.02956)(\text{DEM}35,714,286)}{1.060^{0.5}}+\frac{(0.03151)(\text{DEM}35,714,286)}{1.062}+\frac{(0.02762)(\text{DEM}35,714,286)}{1.060^{1.5}} \\ &+\frac{(0.02568)(\text{DEM}35,714,286)}{1.058^{2}}+\frac{(0.02859)(\text{DEM}35,714,286)}{1.058^{2.5}} \\ &+\frac{(0.02859)(\text{DEM}35,714,286)}{1.058^{3}}+\frac{\text{DEM}35,714,286}{1.058^{3}} \\ &=\text{DEM}35,714,286 \end{aligned}$$

There are four important points here: (1) the floating cash flow at any time is determined by the unannualized interest rate one period earlier; (2) the forward rates determine the expected DEM floating cash flows; (3) the expected DEM floating cash flows are discounted by using spot German interest rates; and (4) the principal of DEM35,714,286 is exchanged at maturity.

Note also that the present value of the expected floating cash flows just computed, DEM35, 714, 286, is the principal value of the swap ($25,000,000/($0.70/DEM)). This is no coincidence. Recall from Section 13.1.2. that the shortcut method of pricing an at-market fixed-for-floating interest rate swap at origination requires the assumption that the present value of floating cash flows will approximately equal the principal value of the swap.

We can conclude that it is important to know and understand why and how to compute the present value of the floating payments. In particular, it is important to be able to know and understand how to value an off-market swap, where, say, a party wishes to pay or receive floating LIBOR plus or minus a margin. Also, it is important for valuing swaps after origination. However, for pricing an at-market fixed–floating currency swap, we can assume that the value of the floating cash flows is equal to the principal value.

Because DEM35,714,286 equals $25 million, the problem now is to compute the fixed U.S. interest rate that makes the present value of the fixed payments equal $25 million. Let this fixed U.S. semiannual rate equal x. The appropriate pricing equation is then

$$\begin{aligned} &\frac{x(\$25,000,000)}{1.045^{0.5}}+\frac{x(\$25,000,000)}{1.055}+\frac{x(25,000,000)}{1.062^{1.5}}+\frac{x(\$25,000,000)}{1.065^{2}}+\frac{x(\$25,000,000)}{1.067^{2.5}} \\ &+\frac{x(\$25,000,000)}{1.068^{3}}+\frac{\$25,000,000}{1.068^{3}}=\$25,000,000 \end{aligned}$$

$$\begin{aligned} &x(24,455,799)+x(23,696,682)+x(22,843,005)+x(22,041,482)+x(21,258,321) \\ &\quad +x(20,522,310)+20,522,310=\$25,000,000 \end{aligned}$$

$$x(\$134,817,600)=\$4,477,690$$

$$x=0.0332$$

Annualizing the semiannual rate of 3.32%, the fixed U.S. rate is 6.64%. If the swap dealer pays the fixed U.S. cash flows, he will offer to pay a rate somewhat below 6.64%, and receive floating DEM. If the swap dealer is the receive-fixed party, he will require a rate slightly above 6.64%, and pay floating DEM.

The FinancialCAD function aaSwp_cpn provides the same answer for valuing this fixed for floating currency swap, as shown in Figure 13.3. The discount rates are computed using $1/[1 + r(0,y)]^y$, where y is the number of years until maturity, assuming a 30/360 day count method.

13.2.3 Valuing a Fixed–Floating Currency Swap After Origination

Suppose that at origination, a fixed–floating currency swap in which fixed U.S. dollars are swapped for floating DEM is priced at 6.64% (fixed U.S. interest rate). The swap tenor is three years, the principal amount is $25 million, payments are semiannual, and the spot exchange rate is $0.70/DEM. This is the swap described in Section 13.2.2.

One year after this swap was originated, *two-year* fixed US $ for floating DEM swaps are being quoted at 7.78% (fixed U.S. interest rate). Also, one year after the swap's origination, spot U.S. interest rates are as follows:

Years to Maturity	Interest Rate
0.50	8.25%
1.00	8.20%
1.50	8.00%
2.00	7.90%

How much is the swap worth for the pay-fixed party (one year after origination)?

Recall from Section 13.1.3 that there are two methods for valuing an interest rate swap after origination. Here, however, there are insufficient data to use the first method to value this currency swap, because German interest data, which are needed to value the remaining floating DEM cash flows, have not been provided. However, there are sufficient data to value the swap by means of the second approach. We will compute the present value of the remaining fixed U.S. dollar payments of the existing swap and compare them to the present value of fixed U.S. dollar payments on a newly originated swap. The value of the swap is the difference between the two. That is, compare the present values of the following two sets of cash flows. The first series of cash flows is the present value of the remaining payments on the old swap:

$$\frac{(0.0664/2)(\$25,000,000)}{1.0825^{0.5}} + \frac{(0.0664/2)(\$25,000,000)}{1.082} + \frac{(0.0664/2)(\$25,000,000)}{1.080^{1.5}}$$
$$+ \frac{(0.0664/2)(\$25,000,000)}{1.079^{2}} + \frac{\$25,000,000}{1.079^{2}}$$
$$= \text{present value of remaining payments on existing swap}$$

$$\$797,745 + \$767,098 + \$739,507 + \$712,911 + \$21,473,217 = \$24,490,478$$

aaSwp_cpn		
Value (settlement) date	24-Sep-2000	
Effective date	24-Sep-2000	
Terminating date	24-Sep-2003	
fair swap value	0	
Structure	1	pay floating and receive fixed
Interpolation method	1	Linear
Exchange of principal	2	exchange of principal
Margin above or below a floating rate	0	
FX spot - pay / receive	0.7	
current fixing of rate as of last reset date	0	
business day convention (see Glossary) - pay leg	1	no date adjustment
direction for date generation - pay leg	1	forward from effective date
cash flow frequency - pay leg	2	semi-annual
odd date list - pay leg	t_32	
accrual method - pay leg	4	30/360
holiday list - pay leg	t_26	
notional principal - pay leg	25000000	
discount factor curve - pay leg	t_43_1	
business day convention (see Glossary) - receive leg	1	no date adjustment
direction for date generation - receive leg	1	forward from effective date
cash flow frequency - receive leg	2	semi-annual
odd date list - receive leg	t_32	
accrual method - receive leg	4	30/360
holiday list - receive leg	t_26	
notional principal - receive leg	35714285.71	
discount factor curve - receive leg	t_43_1	

t_32
odd date list
non-cycle dates on which a cash flow occurs

t_26
holiday list
holiday date

1-Jan-1995

t_43_1
discount factor curve

grid date	discount factor (Germany)	
24-Sep-2000	1	
24-Mar-2001	0.971285862	=1/(1.06)^.5
24-Sep-2001	0.941619586	=1/1.062
24-Mar-2002	0.916307417	=1/(1.06)^1.5
24-Sep-2002	0.893364446	=1/(1.058)^2
24-Mar-2003	0.868532013	=1/(1.058)^2.5
24-Sep-2003	0.844389836	=1/(1.058)^3

t_32
odd date list
non-cycle dates on which a cash flow occurs

t_26
holiday list
holiday date

1-Jan-1995

t_43_1
discount factor curve

grid date	discount factor (United States)	
24-Sep-2000	1	
24-Mar-2001	0.978231976	=1/(1.045)^.5
24-Sep-2001	0.947867299	=1/1.055
24-Mar-2002	0.913720197	=1/(1.062)^1.5
24-Sep-2002	0.881659283	=1/(1.065)^2
24-Mar-2003	0.850332839	=1/(1.067)^2.5
24-Sep-2003	0.820892413	=1/(1.068)^3

par swap rate **0.066425892**

Figure 13.3 The FinancialCAD function aaSwp_cpn.

The second series of cash flows is the present value of the fixed U.S. dollar payments for a new swap:

$$\frac{(0.0778/2)(\$25,000,000)}{1.0825^{0.5}} + \frac{(0.0778/2)(\$25,000,000)}{1.082} + \frac{(0.0778/2)(\$25,000,000)}{1.08^{1.5}}$$
$$\frac{(0.0778/2)(\$25,000,000)}{1.079^{2}} + \frac{\$25,000,000}{1.079^{2}}$$
$$= \text{present value of fixed U.S. dollar payments on a new swap}$$

$$\$934,707 + \$898,799 + \$866,471 + \$835,308 + \$21,473,217 = \$25,008,502$$

Thus, the value of the swap for the pay-fixed party is \$518,024. The pay-fixed party of the existing swap is fortunate to be paying cash flows with a present value of only \$24,490,478, when new swaps would require the payment totaling \$25,008,502.

The FinancialCAD function, aaFixlg_cfx, can solve this problem. It can be used to solve for the present value of the remaining payments on the old swap, and then used to solve for the present value of payments on a newly originated swap. The last line of the spreadsheets in Figure 13.4 shows the present value of the payments that remain to be made on the old swap. You should use aaFixlg_cfx to verify that the present value of the payments on a new swap is \$25,008,502. (*Hint:* Just change the coupon from 0.0664 to 0.0778.) The value of the swap (\$518,024) is the difference between the two present values.

13.2.4 Pricing a Diff Swap

Recall from Chapter 11 that a diff swap is a currency swap in which all payments and receipts are made in one currency. In other words, the notional principal is expressed in one currency. Payments are determined by applying one floating interest rate in one country to that notional principal. Receipts are determined by applying a different floating interest rate from a different country to that same notional principal. If the interest rates in the two countries are different (which they almost certainly will be), a margin is added to one of the two rates. The pricing problem is to determine that margin.

For example, suppose a firm wishes to pay U.S. dollar six-month LIBOR and receive the British pound sterling six-month interest rate, and have both payments denominated in pounds sterling on a notional principal of £100 million. The swap's tenor is two years, and payments are semiannual. Current spot interest rates on zero-coupon debt instruments are as follows:

Years to Maturity	U.S.	Britain
0.5	6.00%	7.00%
1.0	6.60%	7.00%
1.5	6.95%	6.85%
2.0	7.10%	6.75%

The interest data are sufficient to determine forward rates. Define $fr(t1, t2)$ as the six-month forward rate for the period beginning at time $t1$, and ending at time $t2$. Equation (13.2) is used to determine the forward rates:

$$[1 + r(0, t2)]^{t2} = [1 + r(0, t1)]^{t1}[1 + fr(t1, t2)]^{t2-t1}$$

aaFixlg_cfx

Settlement date	24-Sep-2001	
Terminating date	24-Sep-2003	
effective date	24-Sep-2001	
date of first coupon after dated date		
date of last coupon prior to maturity date		
Coupon	0.0664	
Principal	25000000	
cash flow frequency	2	Semi-annual
accrual method for coupons	4	30/360
accrual method for accrued interest	4	30/360
holiday list	t_26	
business day convention (see Glossary)	1	no date adjustment
exchange of principal	3	at maturity
trade position	1	Long
interpolation method	1	Linear
discount factor curve	t_43_1	
output table selection	1	Eight column extended cashflow table (incl date,cpn,npa,total,pvcpn,pvnpa,pvtot,acc)

T_26
holiday list

holiday date
1-Jan-1995

T_43_1
discount factor curve

grid date	discount factor
24-Sep-2001	1
24-Mar-2002	0.961138663
24-Sep-2002	0.924214418
24-Mar-2003	0.890972638
24-Sep-2003	0.858928693

extended cashflow table - aaFixlg_cfx

Date	cash flow amount	Notional principal amount	total cash flow	present value cash flow	present value of the notional amount	total present value of cashflows
24-Mar-2002	830000	0	830000	797745.1	0	797745.1
24-Sep-2002	830000	0	830000	767098	0	767098
24-Mar-2003	830000	0	830000	739507.3	0	739507.3
24-Sep-2003	830000	25000000	25830000	712910.8	21473217	22186128
						24490478

Figure 13.4 Using the FinancialCAD function aaFixlg_cfx to value a fixed–floating currency swap after origination.

Thus, the **unannualized** forward rates are:

	U.S.	**Britain**
$fr(0, 0.5)$	0.029563	0.034408043
$fr(0.5, 1.0)$	0.035390729	0.034408043
$fr(1.0, 1.5)$	0.037561811	0.03223365
$fr(1.5, 2.0)$	0.037069253	0.0317488

Swap dealers will be able to hedge the floating payments by using forward rates, which are the theoretical prices for FRAs. It may be convenient to think that these rates are the expected future spot rates, which will therefore determine the future expected floating payments. However, recall that these forward rates are the expected future spot rates only if the pure expectations theory of the term structure is valid.

Our problem here is to find a margin to add to (or subtract from) the six-month U.S. LIBOR so that the payments, made in pounds sterling, equal the receipts, also made in that currrency. Define x as the margin added to U.S. LIBOR. Also, recall that the time t interest rate typically determines the cash flow at time $t+1$. The present value of the pounds sterling payments (based on U.S. LIBOR) is:

$$\frac{(0.029563+x)(\pounds 100,000,000)}{1.0700^{0.5}}+\frac{(0.035390729+x)(\pounds 100,000,000)}{1.0700^{1.0}}$$
$$+\frac{(0.037561811+x)(\pounds 100,000,000)}{1.0685^{1.5}}+\frac{(0.037069253+x)(\pounds 100,000,000)}{1.0675^{2}}=\text{PV}$$

Expanding:

$$2,857,673+96,673,649x+3,307,545+93,457,944x+3,400,829+90,539,525x$$
$$+3,252,955+87,753,457x=\text{present value of the £ payments based on U.S. LIBOR}$$

Collecting terms:

$$368,424,575x+12,819,002=\text{present value of the £ payments based on U.S. LIBOR}$$

Note that the payments are made in pounds and that the discount rates are the spot British interest rates. However, the payments are made on the basis of U.S. LIBOR plus a margin. Half the U.S forward rates is used because payments are made semiannually.

The (expected) cash flow receipts are determined on the basis of forward British rates. Half of the forward British rates is applied to the notional principal amount. The present value of the pound sterling receipts is:

$$\frac{(0.034408043)(\pounds 100,00,000)}{1.07^{0.5}}+\frac{(0.034408043)(\pounds 100,000,000)}{1.07^{1.0}}+\frac{(0.03223365)(\pounds 100,000,000)}{1.0685^{1.5}}$$
$$+\frac{(0.0317488)(\pounds 100,000,000)}{1.0675^{2}}=\pounds 12,252,236$$

To solve for the unknown margin, x, set present value of the pound payments based on the U.S. LIBOR equal to the present value of the pound receipts:

$$368,424,575x+12,819,002=12,252,236$$

$$x=-0.00153835$$

Thus, 15 basis points will be subtracted from the semiannual U.S. LIBOR. That is, negative 15 basis points is the (semiannual) price of the swap. Each period, the U.S. LIBOR is observed. Then, to determine the pound payment amount at the next payment date, take half the observed interest rate and subtract 15 basis points. The pound cash flow received at time t is determined by taking half the observed British interest rate at time $t-1$.

13.3 PRICING A COMMODITY SWAP

Commodity swaps are priced in the same way as interest rate and currency swaps. Again, the key is to equate the present values of the payments and the receipts. One type of commodity swap is the fixed–floating commodity swap. One party agrees to pay a fixed price at regular intervals for a commodity and the counterparty agrees to pay a floating price. Generally, the notional principal is not exchanged. Rather, the two payments are netted. A commodity swap is equivalent to a strip of forward contracts, each of which has the same forward price. The pricing problem is to solve for that fixed price.

For example, let us find the fixed price for the following fixed-for-floating oil price swap. Suppose the tenor is 12 months with settlement occurring every three months, beginning three months from today. The floating price on each settlement date is the spot price of West Texas Intermediate (WTI), which is a well-known type of crude oil. The problem is to find the fixed price of the swap.

If WTI futures or forward prices are readily available, this is not difficult. Assume that WTI futures prices and spot interest rates are as follows.

Months Until Delivery	WTI Futures Price ($/bbl)	Spot Interest Rate
3	$21.55	4.2%
6	$20.04	4.8%
9	$19.18	5.1%
12	$18.40	5.2%

The swap dealer will use futures contracts to hedge the floating cash flows, so that he can use the futures prices to find the present value of the floating cash flows:

$$\frac{21.55}{1.042^{0.25}} + \frac{20.04}{1.048^{0.5}} + \frac{19.18}{1.051^{0.75}} + \frac{18.40}{1.052} = \$76.87$$

The present value of the fixed cash flows, where x represents the fixed price of the swap, is:

$$\frac{x}{1.042^{0.25}} + \frac{x}{1.048^{0.5}} + \frac{x}{1.051^{0.75}} + \frac{x}{1.052} = x\left(\frac{1}{1.042^{0.25}} + \frac{1}{1.048^{0.5}} + \frac{1}{1.051^{0.75}} + \frac{1}{1.052}\right)$$
$$= 3.88055x$$

By equating the present values of the fixed and floating cash flows, we find that the solution for x is:

$$76.87 = 3.88055x$$

$$x = \$19.81/\text{bbl}$$

Thus, the fixed price of the oil swap is $19.81/bbl. If the swap dealer is paying fixed, he will quote a price a few cents below $19.81/bbl. If the swap dealer is receiving fixed, he will quote a price a few cents above $19.81/bbl.

If forward contracts or futures contracts do not exist on exactly the same commodity underlying the swap, the dealer will use the most similar available commodity to price the swap. In these cases, however, the dealer will most likely widen the bid–asked spread around the theoretical swap price to adjust for risk.

The FinancialCAD function aaSwpcd_bmrk can be used to solve for the theoretical price of a commodity swap, as shown in Figure 13.5.

aaSwpcd_bmrk		
value (settlement) date	24-Sep-2000	
effective date	24-Sep-2000	
terminating date	24-Sep-2001	
fair swap value	0	
structure	1	pay floating and receive fixed
interpolation method	1	linear
cash flow frequency - pay leg	3	quarterly
odd date list - pay leg	t_32	
business day convention (see Glossary) - pay leg	1	no date adjustment
direction for date generation - pay leg	1	forward from effective date
cash flow frequency - receive leg	3	quarterly
odd date list - receive leg	t_32	
business day convention (see Glossary) - receive leg	1	no date adjustment
direction for date generation - receive leg	1	forward from effective date
holiday list	t_26	
notional quantity per period (in units of underlying commodity)	1000	
commodity forward price curve	t_4_3	
spot price per unit of underlying commodity	22	
discount factor curve	t_43_1	

t_32
odd date list
non-cycle dates on which a cash flow occurs

t_32
odd date list
non-cycle dates on which a cash flow occurs

t_26
holiday list

holiday date
1-Jan-1995

t_4_3
commodity forward price curve

expiry date	commodity forward price
24-Dec-2000	21.55
24-Mar-2001	20.04
24-Jun-2001	19.18
24-Sep-2001	18.4

t_43_1
discount factor curve

grid date	discount factor	
24-Sep-2000	1	
24-Dec-2000	0.989767229	=1/1.042^.25
24-Mar-2001	0.976830831	=1/1.048^.5
24-Jun-2001	0.963380747	=1/1.051^.75
24-Sep-2001	0.950570342	=1/1.052

swap - fair benchmark (fixed) price **19.80990517**

Figure 13.5 Using the financialCAD function aaSwpcd_bmrk to solve for the theoretical price of a commodity swap.

13.4 SUMMARY

Pricing swaps is a matter of equating the present value of the payments to the present value of the receipts. Usually, the floating payments are "known" in the sense that the swap dealer will use forward prices or futures prices in their valuation. Then, the pricing problem is usually to solve for the fixed price that makes the present value of the fixed cash flows equal to the present value of the floating cash flows. The discount factors are the yields on zero coupon debt instruments.

Notes

[1]This chapter will ignore the different settlement conventions of forward contracts (covered in Chapter 3) and swaps (covered in Chapter 11).

[2]The assumption that the pure expectations theory is correct is not necessary. Recall from Chapter 5 that market participants can lock in forward rates as their actual future borrowing or lending rates by trading zero-coupon bonds in the spot market. In other words, a swap dealer can make the forward rates be equal to the actual future spot rates they will face as borrowers or lenders, and the "expected" cash flows used in the example can be locked in as actual cash flows.

[3]Although the swap dealer does not know what the future floating payments will be, they can be hedged by engaging in FRAs, which will then effectively fix the floating rates. Similarly, the future floating rates can be locked in by trading spot bonds. Theoretical forward rates for a 12×24 FRA and a 24×36 FRA are 7.0095 and 10.5640%, respectively.

[4]The value of a floating-rate coupon bond may not always exactly equal its par value for three reasons. First, if the issuer becomes more risky (more likely to default) after issuing the bond, the bond will sell at a discount to provide a default risk premium. Second, typically, there will be a lag between the time at which the next floating coupon amount is determined and when it is paid. Third, the rate that determines the coupon payments may not equal the rate used for discounting.

[5]The FinancialCAD function aaSwp_cpn can also be used to price this swap, and to price swaps when there are payments at times less than a year apart (e.g., semiannually or quarterly). However it takes an unusual approach to computing the floating payments (which are based on the forward rates implied in the current term structure). Normally, we would divide the implied forward rates by 2 or 4 to compute semiannual or quarterly cash flows. However, aaSwp_cpn instead solves for the semiannual rate, which if compounded, would give the annual forward rates, and uses them to determine the floating payments (Use the function aaFlleg_cfs to determine floating payments, and you would conclude that they are computed in this way.) The student should solve Problem 13.1 at the end of chapter to learn the processes.

[6]The first payment is determined by the existing one-year LIBOR, at time zero. Each subsequent payment is determined by the interest rate that exists at the start of the period.

[7]The astute reader will recognize that DEM35,714,286×\$0.7/DEM = \$25,000,000. Thus the problem is actually to solve for x in the following equation:

$$x\left[\frac{\$25\text{mm}}{(1.06)^{0.5}}+\frac{\$25\text{mm}}{1.062}+\frac{\$25\text{mm}}{(1.06)^{1.5}}+\frac{\$25\text{mm}}{(1.058)^{2}}+\frac{\$25\text{mm}}{(1.058)^{2.5}}+\frac{\$25\text{mm}}{(1.058)^{3}}\right]+\frac{\$25\text{mm}}{(1.058)^{3}}=\$25{,}915{,}014$$

[8]However, the assumption that these are *expected* cash flows is not necessary. See note 2 in this chapter.

PROBLEMS

13.1 The spot term structure is $r(0, 1/2) = 4.8\%$ (this is the spot six-month interest rate), $r(0, 1) = 5.3\%$, $r(0, 1\,1/2) = 5.7\%$, $r(0, 2) = 6\%$. Compute the price of a plain vanilla fixed–floating interest rate swap with a tenor of two years. Payments are semiannual, beginning six months hence. Use FinancialCAD to solve the problem.

13.2 The spot term structure on August 3, 2000, is

One-year rate = 9.5%

Two-year rate = 9%

Three-year rate = 8.75%

Four-year rate = 8.6%

a. Find the price of a four-year swap with a notional principal of $25 million. The floating rate is one-year LIBOR. Payments are annual.

b. Suppose that one year after the swap in part (a) was originated, the spot term structure has changed to

One-year rate = 6%

Two-year rate = 7.3%

Three-year rate = 7.8%

Four-year rate = 8.1%

What is the value of the swap for the receive-fixed counterparty?

13.3 The spot term structure is $r(0, 1/4) = 7.2\%$, $r(0, 1/2) = 8\%$, $r(0, 3/4) = 7.8\%$, $r(0, 1) = 7.7\%$, $r(0, 1\,1/4) = 7.6\%$, $r(0, 1\,1/2) = r(0, 1\,3/4) = r(0, 2) = 7.5\%$. A firm approaches a swap dealer and requests that an off-market swap be priced in which the firm is the pay-fixed counterparty. The firm wishes to pay 6.5% fixed. Assume a national principal of $25,000,000. What payment will the swap dealer request? Who will make the payment, the firm or the swap dealer? Why?

13.4 A firm enters into a plain vanilla fixed–floating interest rate swap with a swap dealer. Here are the terms:

Notional principal	$50 million
Fixed rate paid by firm	8%
Floating rate received by firm	Six-month LIBOR
Tenor	Three years
First payment	Six months after origination
Subsequent payment dates	Every six months, up to and including the termination date
Day count method	30/360

The terms of the swap require that it be marked to market every year. One year after the swap's origination (immediately after the second payment), otherwise equivalent new swaps with a tenor of two years are priced with a fixed rate of 7%. How much will the mark-to-market cash payment be? Who will pay this amount? Use the FinancialCAD function aaSwpi to solve this problem; note that your answer obtained through FinancialCAD will differ from your hand-calculated solution as a result of the process discussed in note 5. Assume the spot rates one year after the swap's originations are: $r(0, 0.5) = 6\%$; $r(0, 1) = 6.30\%$; $r(0, 1.5) = 6.90\%$, $r(0, 2) = 7.10\%$.

13.5 Consider the plain vanilla swap in Section 13.1. Suppose that two years after the swap was originated, the pay-fixed party must mark the swap to market. The remaining tenor of the swap is only two years. At that date (two years after origination), the term structure of interest rates is $r(0,1)=7.5\%$, $r(0,2)=7\%$, and $r(0,3)=6.8\%$. What is the value of the swap for the pay-fixed party? Also use Financial-CAD to solve this problem.

13.6 Rework out the currency swap pricing problem in Section 13.2.1 assuming that the principal amounts are **not** exchanged at the swap's maturity.

13.7 A firm wishes to enter into a fixed–fixed currency swap. The principal amount is \$60 million, and the firm wishes to receive a fixed rate of 7.5% in U.S. dollars. The firm will pay Japanese yen. Suppose the spot exchange rate is ¥98/\$. The swap tenor is two years, and payments will be quarterly. Principal amounts will be swapped at origination and at termination. Spot interest rates on zero coupon debt instruments in the U.S. are as follows:

Years to Maturity	Interest Rate
0.25	5.25%
0.50	6.00%
0.75	6.60%
1.00	7.00%
1.25	7.25%
1.50	7.40%
1.75	7.50%
2.00	7.55%

Forward rates in Japan, defined as $fr(t1, t2)$, where the forward period begins at time $t1$ and ends at time $t2$ are as follows:

$fr(0, 0.25) = 0.80\%$
$fr(0.25, 0.50) = 0.88\%$
$fr(0.50, 0.75) = 1.0\%$
$fr(0.75, 1.00) = 1.2\%$
$fr(1.00, 1.25) = 1.29\%$
$fr(1.25, 1.50) = 1.35\%$
$fr(1.50, 1.75) = 1.42\%$
$fr(1.75, 2.00) = 1.45\%$

Compute the fixed Japanese interest rate for the swap.

13.8 Use the data in Problem 13.7 to find the fixed U.S. interest rate that prices a fixed–floating currency swap. Fixed U.S. dollars will be swapped for floating yen.

13.9 Given the price determined in the diff swap in Section 13.2.4., suppose that six months after the swap was originated, six-month U.S. LIBOR is 6.2% and six-month British LIBOR is 7.9%. What is difference check amount? Who will pay it and who will receive it?

13.10 A firm wishes to pay quarterly yen LIBOR and receive quarterly U.S. dollar LIBOR, and have both payments denominated in U.S. dollars on a notional principal of \$60 million. This diff swap's tenor is two years, and payments are quarterly. Current spot interest rates on zero coupon debt instruments are as follows:

Years to Maturity	U.S.	Japan
0.25	4.5%	1.0%
0.5	5.0%	1.2%
0.75	5.4%	1.6%
1	5.8%	1.8%
1.25	5.9%	2.0%
1.5	6.0%	2.3%
1.75	6.1%	2.5%
2	6.2%	2.7%

Determine the price of this swap.

13.11 Find the fixed price for a fixed-for-floating gold price swap. Settlement will be every six months, beginning four months from today. The tenor will be 22 months. The floating payment will be based on the spot price of gold on each settlement date. Today's gold futures prices are:

Delivery_months hence	gold futures price
4	409.20
10	415.10

16	420.40
22	425.80

Spot interest rates for zero coupon bonds are $r(0,t)$, where t is the number of months until the bond matures are $r(0,0.33)=4\%$, $r(0,0.83)=4.2\%$, $r(0,1.33)=4.5\%$, $r(0,1.83)=4.7\%$. Find the price (the fixed price) of the swap. Also use FinancialCAD to check your solution.

13.12 A mutual fund owns a portfolio of German stocks. It wishes to enter into an equity index swap in which it agrees to pay the rate of return on the DAX index (the DAX is a German stock index), plus or minus a margin, and receive German 3-month LIBOR. The tenor will be one year, and payments will be quarterly, beginning 3 months hence. A payment at time t will be made on the basis of the actual rate of return on the stock index during the period ending at time t, and the interest rate at time t. The notional principal is €80 million. The day count method for the interest rate is 30/360. The swap dealer observes the following futures prices:

Delivery months hence	DAX futures	German LIBOR futures
3	2560	7.0%
6	2598	6.9%
9	2633	6.8%
12	2660	6.6%

At origination, the DAX index is at 2,540, and spot 3-month German LIBOR is 7.1%. Compute the proper margin that the swap dealer will add or subtract to the DAX index. Why would the mutual fund wish to engage in this swap? Why would the dealer?

13.13 A firm has a floating-rate liability. At time t, it pays whatever six-month LIBOR was at the start of the period. The liability matures in two years, and interest payments are made every six months. The firm believes that interest rates are about to rise.

a. How can the firm use a swap to hedge against the risk that interest rates will rise?

b. How can the firm use a strip of FRAs to hedge against the risk that interest rates will rise? If the 6×12 FRA is quoted by a dealer at 7.3/7.4, which rate is appropriate for the firm?

c. Which should be used, the swap or the strip of FRAs? Suppose that spot six-month LIBOR is 7%. The swap dealer will receive 8% or pay 7.9% against six-month LIBOR. FRAs are quoted as follows:

6×12	7.3/7.4
12×18	8.1/8.22
18×24	8.42/8.54

Should the firm use the swap or the FRA?

13.14 In Section 13.1.4, the IRR for a series of uneven cash flows is reported as 1.5214%. Use Excel to compute this IRR. The present value in June 1999 is $50 million. The cash flows are as follows:

Borrowing Period Starting	Floating Payment
Sept 1999	$680,625
Dec 1999	$726,250
Mar 2000	$726,875
June 2000	$756,250
Sept 2000	$779,375
Dec 2000	$808,125
Mar 2001	$806,250
June 2001	$813,750

13.15 On April 28, 2000, a firm wants to use Eurodollar futures prices to compute the theoretical price of a plain vanilla interest rate swap. The tenor of the swap is three years, and payments will be made quarterly. The first payment will be made in September 2000. The notional principal of the swap is $60 million.

Here are the Eurodollar futures prices that existed on April 28, 2000:

Expiration Month	Futures Price (IMM Index)
May 2000	93.39
June	93.25
July	93.15
August	93.04
September	92.96
December	92.75
March 2001	92.71
June	92.67
September	92.65
December	92.62
March 2002	92.69
June	92.70
September	92.71
December	92.66
March 2003	92.60

Find the theoretical price of the swap.

PART 4

OPTIONS

CHAPTER 14

Introduction to Options

To this point, we have presented three classes of risk management instruments: forward contracts, futures contracts, and swaps (which can be viewed as a portfolio of forward contracts). These three classes of risk management instruments share two important features. First, both parties to these contracts, (i.e., the *long* and the *short*) are initially bound by the terms of the contract. Second, there is no cash payment required by one party from the other at the initiation of the contract.

In this chapter, we introduce a new class of risk management instruments known as options. There are a number of new terms, concepts, and institutional details associated with options. From a risk management standpoint, a mastery of these fundamental definitions and ideas provides a basis for understanding how options can be used to manage risk.

Perhaps the most fundamental option concept is that option contracts separate rights from obligations. That is, the *long* has rights, whereas the *short* has obligations. As a consequence of this separation, the long must pay the short a dollar amount at the initiation of the option contract. This dollar amount, called either the option price, the option premium, or the option value, is the subject of Chapters 17 and 18.

Depending on the nature of the rights and obligations in the option contract, options are classified into two categories known as call options or put options. We begin the study of options by examining the features of call options.

14.1 Call Options

A call option is a contract that gives its owner the right, but not the obligation, to buy something at a specified price on or before a specified date. The "something" that can be bought with the option is called the **underlying asset**. In this book, the price of the underlying asset will be usually denoted as an S. Much of the description of options in this text will involve options on 100 shares of a specific firm's stock. For example, if you buy one call option on IBM today, you have the right to purchase 100 shares of IBM's common stock at a specified price, any time between today and a prespecified date.

The fact that the call owner does not have the obligation to exercise the option means that he has limited liability. Should the price of the underlying asset fall, he can just walk away from the call contract without ever having had to acquire the underlying asset.

The specified price is called the **exercise price**, the **strike price**, or the **striking price**, and in this text, it will be denoted K (in other books and articles it may be denoted X, or in some cases, E).

The **expiration date** of an option, or the time to expiration, will be denoted T in this text. The expiration date of U.S. exchange listed options on 100 shares of common stock is the Saturday immediately following the third Friday of the expiration month. The last trading day is the third Friday of the expiration month. Some **index options**[1] expire on other dates, and **futures options**[2] expire on a wide range of dates. Finally, **OTC options**,[3] which are custom-made contracts that do not trade on any exchanges, and **flex options**,[4] which are custom-made contracts used by institutional traders, may expire on any day. Investors in options should always ascertain the expiration dates of contracts they are trading.

The general option definition just given applies to **American-style** options. The owner of an American option can exercise it *on or before* the expiration date. The owner of a **European-style** option can exercise it only on the expiration date. The geographic designation, however, is largely irrelevant. Most options that are actively traded in the world (including in Europe) are American-style options. Because American options give the owner an additional timing option (the right, but not the obligation, to exercise early), they cannot be worth less than otherwise equivalent European options. Note, however, that the holder of any traded option can sell the option at any time before expiration.

Most likely, when you think of an option, you think of a call option on 100 shares of an underlying stock. However, other actively traded options exist on stock market indices, futures contracts, foreign exchange, and debt instruments. The OTC interest rate option market is tremendous. Options on other assets can also be created. Companies acquire options to buy other companies. Real estate developers purchase options to buy land. A professional sports figure might negotiate a contract that gives his team an option to utilize his services in the last year of the contract at a specified salary; if the team decides not to exercise the option, it pays the ballplayer a small amount, thereby freeing him to offer his skills to any other team. A **warranty** is an option to return the good to the manufacturer of the item or the store in which it was purchased.

A call option's price, frequently referred to as the option **premium**, will be denoted C. Frequently, time subscripts will be necessary for S, the underlying asset, and for C. The subscript 0 will be used in reference to "today." Thus, C_0 is today's call price, and S_0 is today's stock price. Two other time subscripts often used are T for the option's expiration date, and t for some general date between today and expiration.

Option contracts are created when a buyer and a seller (the seller is frequently referred to as the option **writer**) agree on a price. The buyer pays the premium to the seller. If the call owner can exercise the call, thereby calling away the underlying asset, the call writer must deliver it. In other words, the writer of a call has the obligation to sell the underlying asset to the call owner at the strike price, should the owner decide to exercise. The call writer then receives $\$K$ in exchange for the underlying asset.

The call writer might not own the underlying asset. In this case, the arrangement is referred to as a **naked call**. If the owner of a call exercises his option and if the exercise is assigned to a naked writer, the naked writer must buy the underlying asset at its prevailing market price, which will then allow him to sell the asset at the strike price to the exerciser of the call. When an individual writes a naked call, brokers will want a guarantee that the capital is available to buy the underlying asset if necessary. Margin requirements exist as a form of collateral to ensure that the writer of a naked option can fulfill the terms of the contract. The term **covered call** is applied when the underlying asset is owned when the call is sold.

Call buyers hope that the price of the underlying asset will increase in value. An investor profits if, say, he owns a call to buy 100 shares of IBM at a strike price of $120 per share and IBM rises from $120/share to $125/share soon after the purchase of the call. All else equal, an option to buy IBM at $120 is certainly more valuable when IBM is selling for $125 than when it sells for $120. Thus, all else equal, the call option increases in value as the price of IBM rises. The writer of the naked call hopes IBM shares will tumble or, at the very least, will not exceed their price level at the time the option was written, for under these circumstances the writer profits.

On any day, option owners may take one of three actions regarding their positions:

a. Do nothing.

b. Close out the position in the options market. Thus, one who owns a publicly traded option (i.e., is long the option) can sell it at the market price. Someone who has previously written an option can buy it back. A nontraded option can be effectively offset when its owner sells an option with terms equivalent to the one he owns.

c. The owner of an American option can exercise it. In this case, the call seller must deliver the underlying asset. The exerciser of the call then pays the strike price to an investor who earlier wrote the call, and was assigned the exercise.

If the call owner has done nothing by the close of trading on the call's expiration day, one of two things will happen

a. If the price of the underlying asset is less than the strike price, the call expires worthless. For example, if $K=40$, and the expiration day stock price is $S_T=37$, the call owner will not exercise the call; because if he did, he would have to pay $K=\$40$ to acquire the asset, but the asset price in the spot market is only $37/share. In this case, $C_T=0$ and option exercise is irrational.

b. If the price of the underlying asset exceeds the strike price, then ignoring such "market imperfections" as transactions costs and taxes, the call owner would be irrational not to exercise. After all, the call owner has the right to pay $\$K$ to buy something that is worth more than $\$K$. If the call owner has not closed the position or exercised it by the last trading day, his brokerage house may exercise the option for him. An investor must sign an "option agreement" before beginning to trade options, and such contracts normally spell out what the brokerage house will do if an investor fails to exercise an option when it would be rational to do so ($S_T > K$ for a call). Note that some brokerage houses demand that option owners provide notification that they intend to exercise **before** the close of trading on the last trading day.

14.2 Put Options

The owner of an American put option has the right, but not the obligation, to sell something at a specified strike price, on or before a specified expiration date. A European put can be exercised only on its expiration date. As with call options, however, owners of traded put options can sell them before expiration.

The seller of a put has the obligation to take delivery of the underlying asset, should the put owner decide to exercise his option. The put seller must then pay $\$K$ for the underlying asset. When a put contract is created, the put buyer pays the put premium to the put seller. If the price of the underlying asset rises above the strike price, and stays there, the put expires worthless. The put seller gets to keep the premium as a profit, and the put buyer suffers a loss.

Buying puts is a bearish strategy.[5] That is, put buyers believe that the price of the underlying asset will fall. The price of the underlying asset must fall below the strike price for the put to have value at expiration. Selling puts is a bullish strategy. Put sellers believe that the price of the underlying asset will be above the strike price at expiration.

14.3 In the Money, At the Money, Out of the Money

At any time t, an option may be **in the money**, **at the money** or **out of the money**. A call is said to be in the money if the current stock price is greater than the strike price. A call is at the money when the stock price equals the strike price, and out of the money when the stock price is less than the strike price. If $S_t >> K$ (the stock price is "much" higher than the strike price), the call is deep in the money, or way in the money. If $S_t << K$, the call is deep out of the money. If the stock price is close to K, the appropriate jargon is that it is near the money, at the money, just out of the money, or just in the money. The terms are reversed for puts. Thus, if the stock price is below the strike price, the put is in the money. If $S_t > K$, the put is out of the money.

Table 14.1 summarizes these terms.

14.4 Intrinsic Value and Time Value

At all times before expiration, any option premium can be split into two parts: **intrinsic value** (sometimes called parity value) and **time value** (sometimes called premium over parity). The intrinsic value of a call is the amount the option is in the money, if it is in the money. If the call is at or out of the money, its intrinsic value is zero. Thus,

$$\text{intrinsic value of a call} = \begin{cases} S_t - K & \text{if } S_t > K \\ 0 & \text{if } S_t < K \end{cases}$$

Another way of denoting this is:

$$\text{intrinsic value of a call} = \max[0, S_t - K]$$

which is read: "The intrinsic value of a call is the greater of 0 or $S_t - K$."

The time value of a call is the difference between its premium and its intrinsic value. Therefore, a call that is out of the money or at the money has only time value. Usually, the maximum time value exists when the call (or put, for that matter) is at the money. A call that is in the money may or may not have time value. It will have no time value if $C_t = S_t - K$. An in-the-money call will have time value if $C_t > S_t - K$. The longer the time to expiration, the greater a call's time value, all

TABLE 14.1 In the Money, Out of the Money, and At the Money

	Calls	Puts
In the money	$S_t > K$	$S_t < K$
Out of the money	$S_t < K$	$S_t > K$
At the money	$S_t \cong K$	$S_t \cong K$

else equal.[6] At expiration, a call will have no time value and will sell for either 0 (if $S_T < K$), or $S_T - K$ (if it finishes in the money, in which case $S_T > K$).

Before expiration, the time value of a call is:

$$\text{time value of a call} = C_t - \{\max[0,\ S_t - K]\}$$

Similar concepts exist for puts:

$$\text{intrinsic value of a put} = \begin{cases} K - S_t & \text{if } S_t < K \\ 0 & \text{if } S_t > K \end{cases}$$

That is, the intrinsic value of a put can be stated as $\max[0, K - S_t]$. The premium for a put consists only of time value if it is out of the money or at the money. If the put is in the money, it may or may not have time value:

$$\text{time value of a put} = P_t - \{\max[0, K - S_t]\}$$

14.5 Payout Protection

Few, if any, traded options are protected against regular cash dividend distributions. Thus, suppose a stock trades ex-dividend.[7] The value of the stock should fall by about the amount of the dividend.[8] It is logical that the stock's value should fall by the ex-dividend amount. Consider this: on the day before the ex-date, the stock price should equal the per share value of all the firm's assets, after debt claims have been paid. On the ex-date, the firm's assets have changed: they have been reduced in value by the amount of the cash dividend. Thus a stock's price should fall by the ex-dividend amount on the ex-dividend day.[9]

Option contract terms are not affected by dividend payments or by announcements that the firm will make a future payment. This is what is meant by payout *un*protected. But, the stock price will nonetheless likely fall on the ex-date. Knowing that the firm will trade ex-dividend in the near future (before the option's expiration date) has the effect of making calls less valuable, and puts more valuable. Owning a call is a price appreciation strategy, and ex-dividend days usually result in price declines. However, owners of puts want the prices of the underlying assets to decline, and ex-dividend days therefore make puts more valuable, all else equal.

Listed option contract terms are, however, adjusted for stock distributions such as stock dividends and stock splits.[10] For a stock split that maintains round, hundred-share lots (e.g., a 2-for-1 or 3-for-1 split), the calls split, too. For example, suppose you own a call on a stock currently selling for \$60/share; the strike price is \$50. Now assume the stock splits 2 for 1. The stock price will fall to about \$30, and you will own 2 calls with \$25 strike prices. Each call will cover 100 shares of the post-split stock.

Option contract terms are adjusted in other ways when there are odd-sized splits (e.g., 3 for 2), or stock dividends (e.g., a 10% stock dividend). Most frequently, the strike price is adjusted, as are the number of shares underlying the contract. For example, consider the call with a \$50 strike price. Now assume the stock has a 10% stock dividend. The call contract's terms will be adjusted to reduce the strike price by 10% to \$45, and the contract will cover 110 shares of stock.

When a firm is merged into an existing firm, is taken over by another firm, or spins off a subsidiary firm, option terms will again be adjusted.

14.6 Pricing at Expiration

In our discussion of pricing at expiration, we will assume that there are no transactions costs of any kind. In other words, investors can buy and/or sell stock and options at a single price (there are no bid–ask spreads or price pressure caused by the trade), with no commissions, and they can do so instantaneously. Furthermore, we assume that investors can trade as many options as they want at the quoted price. In reality, markets are not like this, though the conditions are approached for some market participants.

14.6.1 Call Values at Expiration

If the stock price at expiration is less than or equal to the strike price, the call has finished out of the money and is worthless. This means that if $S_T \leq K$, then $C_T = 0$. No one would be willing to buy an option to buy something today at a (strike) price that exceeds the (market) price. If an out-of-the-money call did sell for a positive price at expiration (or one second prior to expiration), an investor could earn an arbitrage profit by selling it, and receiving the premium (a cash inflow). Then, assuming the option is not exercised (and no rational person should ever exercise out-of-the money options), the call writer keeps the premium. If a call owner irrationally exercised the out-of-the-money call, the call writer would buy the underlying asset for S_T and deliver it for K; since $S_T \leq K$, the call writer would receive yet another cash inflow. In other words, if $C_T > 0$ when $S_T \leq K$ an investor could have a positive cash flow by writing the call and a nonnegative cash flow an instant later. This is arbitrage, which should not exist in well-functioning markets.

If the option finishes in the money, it will be worth its intrinsic value. For a call,

$$C_T = \begin{cases} S_T - K & \text{if } S_T > K \\ 0 & \text{if } S_T \leq K \end{cases}$$

That is, $C_T = \max\,[0, S_T - K]$, which says that C_T is the greater of 0 and $S_T - K$.

We can prove that an in-the-money call must sell for exactly $S_T - K$ at expiration. In the remainder of this book, several proofs of propositions will be made. The method of proof will almost always be the same. First we will ask: "What if the proposition is violated?" If it is violated, we can create an inequality in the form $A > 0$. The trades implied by A will then be made, leading to a cash inflow. An "arbitrage table" will then be constructed to demonstrate that there is a cash inflow on at least one date, and positive or zero cash flows at all other dates.

So, suppose $C_T < S_T - K$?

Then, $C_T - S_T + K < 0$.

This means that $-C_T + S_T - K > 0$. Note that this is an inequality in the form $A > 0$, where $-C_T + S_T - K$ is expression A. Table 14.2a presents an arbitrage table that proves the proposition.

TABLE 14.2a Arbitrage Table for $-C_T + S_T - K > 0$

Transaction Today	Cash Flow
Buy call	$-C_T$
Exercise call to acquire stock	$-K$
Sell stock	$+S_T$
	>0

EXAMPLE 14.1 Suppose that a call with a strike price of $K=40$ is selling for $C_T=$ 3 3/4 at expiration, when the stock is selling for $S_T=44$. To arbitrage, an individual could buy the call on 100 shares of stock for \$375. Then he will exercise the call, and acquire the 100 shares of stock for \$4000. Finally, he will sell the shares in the spot market for \$4400. The arbitrage profit is \$25.

EXAMPLE 14.2 Suppose that $C_T=4\,1/8$ when $S_T=44$ and $K=40$. An arbitrageur can sell the call on 100 shares of the stock, thereby receiving \$412.50, and buy the stock for \$4400. When the in-the-money call is exercised, he delivers the shares and receives \$4000. The arbitrage profit is \$12.50.

You can see that the strategy of purchasing a call, exercising it immediately to acquire the stock, and then selling the stock generates a positive cash flow at no risk. This is an example of arbitrage.

To finish the proof that $C_T=S_T-K$ if $S_T>K$, ask:

What if $C_T>S_T-K$?

Then, $C_T-S_T+K>0$.

Table 14.2b summarizes the arbitrage transactions that an investor can undertake to exploit this mispricing.

TABLE 14.2b Arbitrage Table for $C_T-S_T+K>0$

Transaction Today	Cash Flow
Sell call	$+C_T$
Buy stock	$-S_T$
Deliver stock (if exercised, as it should be, if it is in the money)	$+K$
	>0

Example 14.2 illustrates how an arbitrage profit can be realized if $C_T>S_T-K$.

The pricing of a call at expiration can be summarized in a simple diagram. The value of a call at expiration is a function only of the price of the stock on that date, relative to the strike price. Figure 14.1 shows that if $S_T<K$, the call has finished out of the money and is worthless. This is the horizontal line segment at $C_T=0$, to the left of K. If the stock price at expiration exceeds the strike price, the call is worth the difference between the two, or $C_T=S_T-K$. This is shown as the diagonal line that rises from $S_T=K$ at a 45° angle.

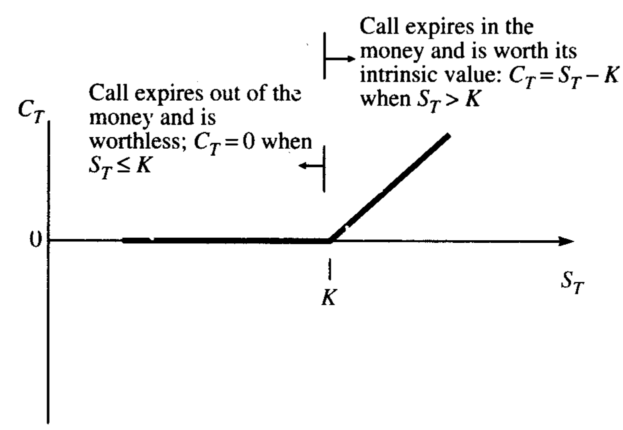


Figure 14.1 Call value at expiration.

14.6.2 Put Values at Expiration

If the put finishes out of the money, the put is worthless because $S_T > K$. To prove this, ask: What would an investor do if $S_T > K$ and $P_T > 0$? He would sell the put just before expiration, and receive the premium. This is a cash inflow. A rational put owner would never exercise an out-of-the-money put. To do so would mean selling something to a put writer for \$$K$ when it could have been sold for a higher price in the market for \$$S_T$. Even if an irrational owner did exercise, the investor who sold the out-of-the-money put at expiration would profit even more, because he would pay \$$K$ for the asset and immediately resell it for the higher price, \$$S_T$.

A put that finishes in the money must sell for $K - S_T$. To prove this, first ask:

What if $P_T > K - S_T$?

Then, $P_T - K + S_T > 0$.

Table 14.3a is the arbitrage table that shows the steps that would provide an arbitrage profit.

Suppose $P_T < K - S_T$?

Then, $P_T - K + S_T < 0$, or $-P_T + K - S_T > 0$.

Table 14.3b presents the arbitrage table for the situation in which $P_T < K - S_T$ exists.

Figure 14.2 depicts the expiration day pricing of puts. If $S_T > K$, the put finishes out of the money and is worth zero. If the stock price closes below \$$K$ per share, the put has an intrinsic value of

TABLE 14.3a Arbitrage Table for $P_T - K + S_T > 0$

Transaction Today	Cash Flow
Sell put	$+P_T$
The put will be exercised, acquire stock	$-K$
Sell stock	$+S_T$
	>0

TABLE 14.3b Arbitrage Table for $P_T - K + S_T < 0$

Transaction Today	Cash Flow
Buy put	$-P_T$
Buy stock	$-S_T$
Exercise put	$+K$
	>0

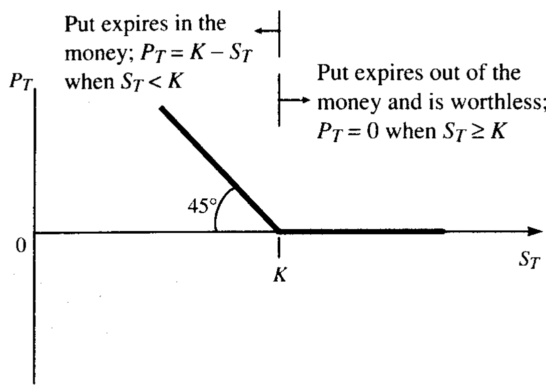


Figure 14.2 Put value at expiration.

$P_T = K - S_T$, and this is shown as the diagonal line with a slope of -1 that originates on the x axis at K. The highest value a put can attain occurs if $S_T = 0$, in which case the put is worth $\$K$ at expiration. Summarizing, we write

$$P_T = \begin{cases} K - S_T & \text{if } S_T < K \\ 0 & \text{if } S_T \geq K \end{cases}$$

Stated another way, P_T is the greater of 0 and $K - S_T$, or $P_T = \max[0, K - S_T]$.

14.7 A Brief Look at Option Pricing Before Expiration

14.7.1 Calls

Before expiration, calls will *usually* sell for at least their intrinsic value.[11] That is, before expiration, calls may or may not have time value. Each curve in Figure 14.3a illustrates how C_t is a function of S_t for different times to expiration. A call that has several months until expiration might sell according to curve a in Figure 14.3a. Given the strike price that is indicated K, and any stock price at a time $t1$ months prior to maturity, S_{t1}, curve a shows how a typical call would be valued as a function of the stock price. As time passes, so that there are $t2$ months until maturity, with $t2 < t1$, the pricing curve gradually moves to a position depicted as curve b. As more time passes, the pricing curve moves toward curve c. At expiration, curve c is the pricing curve, the same as depicted in Figure 14.1.

Consider Figure 14.3b. In this example, the call has a strike of 40, and curve *a* prices a call with four months to maturity. If the stock price at that date is 35, the call might sell for \$2.50 (\$250 for a call on 100 shares). If the stock price suddenly jumped to \$40 per share, the call might sell for \$5 (all else equal). If the underlying stock sold for \$45/share, the call might be worth \$8.375. As time passes, the value of the call will decline, all else equal. Thus, if curve *b* is for one month to maturity, the call might be worth only \$0.50, \$2.375, and \$6 at stock prices of \$35, \$40, and \$45, respectively. Options are frequently termed "decaying," or "wasting" assets because they lose value as time passes, all else equal.[12]

Thus, we have alluded to three variables that contribute to a call's value prior to its expiration date: the stock price, the strike price, and the time to expiration (S_t, K, and T, respectively). There are at least three other variables that affect call values. They are the variance of the stock's distribution of returns (volatility), the interest rate, and the dividend amounts on any ex-dividend dates between the current date and the expiration date.[13]

The next step is to develop some intuition about whether call values should increase or decrease in the event of a change in any of these six variables, all else equal. Eventually a model will be introduced that will provide us with a theoretical value of an option: the Black–Scholes

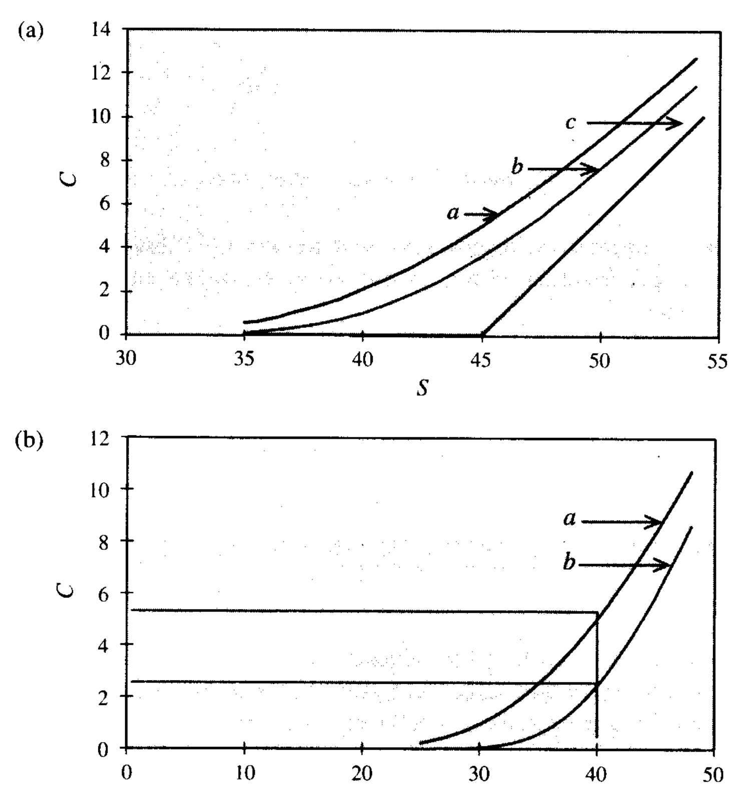


Figure 14.3 Call option values are a function of the stock price and the time to expiration. The call depicted by curve *a* has the longest time to expiration. Curve *c* has no time to expiration. In Figure 14.3b, The at-the-money call decays in value from \$5 to \$2.375 as three months pass.

option pricing model. This model is a tool that can more precisely predict how option values will change, given a small change in any one of the parameters of the model, and holding all other parameters equal. Those familiar with differential calculus will recognize that we are referring to "partial derivatives."

Based on earlier discussion, we predict that a European call's value will be higher when:

a. The strike price K is lower.

b. The stock price S is higher.

c. The time to expiration T is longer (assuming no dividends).

Statement *a* is intuitive because an investor would like to pay as little as possible for the underlying asset, should he exercise the call. Statement **b** is true because the higher the stock price, the more valuable the right to acquire it at a fixed price, K. Finally, statement **c** ought to be intuitive: Would you rather have the right to acquire something during the next month or during the next year? A call option with a longer life provides more time to realize the favorable outcomes that lead to profits for the call owner.

For the other three important determinants of call values, we predict that a call's value will be higher when:

d. The volatility σ of the underlying asset is higher.

e. The level of interest rates r is higher.

f. The dollar dividends to be paid on ex-dividend dates prior to expiration are smaller.

Mathematically, these relations are denoted as follows for European call options:

$$\frac{\partial C}{\partial K} < 0, \quad \frac{\partial C}{\partial S} > 0, \quad \frac{\partial C}{\partial T} > 0, \quad \frac{\partial C}{\partial \sigma} > 0, \quad \frac{\partial C}{\partial r} > 0, \quad \text{and} \quad \frac{\partial C}{\partial \text{div}} < 0$$

The partial derivative notation denotes how a call's value changes when one of its determinants changes by a small amount, all else equal. For example the notation $\partial C/\partial S > 0$ means that when the price of a stock rises a small amount, the value of a call on the stock is predicted to rise, with all else (K, T, σ, r, div) held unchanged. These partial derivatives are widely used by options traders, and some have been named. For example $\partial C/\partial S$ is a call's **delta**, and $\partial C/\partial T$ is a call's **theta**.

We understand intuitively that calls increase in value when the volatility of the underlying asset increases because if the variance of possible future stock prices increases, the call owner will likely benefit if the stock happens to increase by a large amount. It is true that if the variance increases, there is also a greater probability of a price decline. But call owners do not care how much the call is out of the money at expiration. The call is worthless at expiration if S_T is just one-eighth of a point below K, or 8 points or more below K. Call owners profit from the wider range of expiration day stock prices above K, but do not lose from the wider range of possible stock price declines below K. Hence, calls are worth more when the variance of returns for the underlying asset is greater.

Concerning the increase in call values as interest rates rise, consider the purchase of a call as a substitute for actually buying the underlying asset. We will soon see that calls will always sell for less than the underlying asset itself. Thus an investor can buy the stock, or buy a call for a fraction of the stock's price and invest the remainder in riskless interest-bearing assets (e.g., Treasury bills). The higher the level of interest rates, the more the latter alternative will be preferred, since more can be earned on the investment in T-bills. Thus, calls are worth more as interest rates rise, all else equal.

There is another intuitive explanation for the sign of $\partial C/\partial r$ that will become clearer when the binomial option pricing model and the Black–Scholes option pricing model are discussed. We will see that under certain assumptions, if an investor buys a fraction X $(0 \leq X \leq 1)$ of a share of stock and sells 1 call, he will have created a riskless position that will earn the riskless rate of return. This portfolio will provide the same payoff no matter what comes to pass at the next date. All else equal, if interest rates rise, a higher riskless rate of return must be earned on the portfolio. This can occur only with a higher call price, which will lower the initial investment in the stock plus written call portfolio. Thus, if interest rates rise, call values must rise.

Finally, we have already discussed how stock prices fall on ex-dividend days. Thus, the greater the dividend amounts on ex-days prior to the expiration day of the call, the less calls will be worth.

14.7.2 Puts

The pricing of puts prior to expiration might follow the process depicted in Figure 14.4.[14] Curve *a* shows put values for a range of stock prices when there is a great deal of time until expiration. Curve *b* is for a shorter times to expiration, and *d* illustrates expiration day pricing of puts. The situation represented by curve *d* corresponds to Figure 14.2. The slope of each curve ranges from almost 0 at the far right (deep out-of-the-money puts) to −1.0 at the far left (deep-in-the-money puts).

It should be intuitive that put values increase as the strike price increases, and as the price of the underlying asset declines. Puts are more valuable the more cheaply the owner can purchase the underlying stock, and also put the shares to someone at a higher strike price.

The influence of time passage on put prices, all else equal, is not so intuitive, though. If there is more time to expiration, then there is a greater range of possible stock prices that can exist at expiration.[15] This works to increase put values. However, the purchase of a put is also a substitute for selling the stock. If an investor sells the stock today, the proceeds are received today. If, instead, a put is purchased, the proceeds from the stock sale are not received until the put has been exercised, possibly as late as the expiration date. All else equal, an investor would rather have the proceeds today than at the later expiration date. The longer the time to expiration, the less valuable is the receipt of the proceeds from the put's exercise. In other words, the present value of $K is

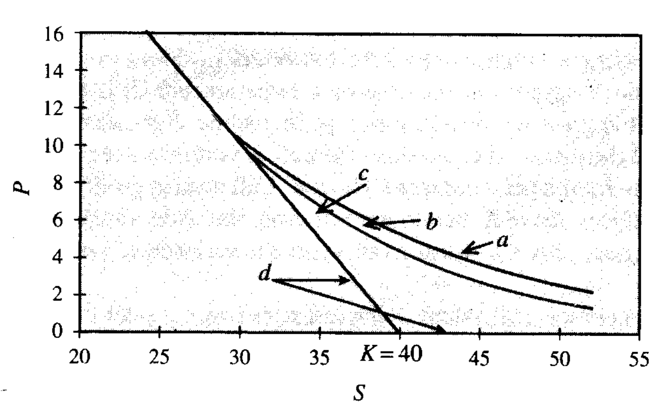


Figure 14.4 Put pricing prior to expiration. Curve *a* is for a put with a long time to expiration. The time to expiration for put option *b* is shorter than that of *a*. The option of curve *d* has no time to expiration.

less, the longer the time to expiration. Thus, the value of the put is less, when viewed as a substitute for selling the stock today. In sum, these two competing influences (the greater range of possible stock prices vs the delayed receipt of the $K proceeds) make for an indeterminate prediction for how puts change in value as time passes.[16]

Greater volatility of the underlying asset increases the value of a put. A greater range of future stock prices increases the chance of high payoffs. The effect becomes less important, however, as the price of the underlying asset declines. At very low stock prices, there is not much range for further price declines. Thus, the impact of volatility on put prices declines as the price of the underlying asset declines.

The same argument about the timing of the receipt of $K applies to the prediction that as interest rates rise, put values decline, all else equal. The purchase of a put is a substitute for the sale of the stock. An investor gets the proceeds from the cash sale of the stock immediately but must wait to receive the $K until exercising the put. At high interest rates, an investor would rather sell the stock today to be able to get the use of the proceeds today, rather than wait to get the strike price at expiration; holding onto in-the-money puts thus becomes less attractive at high interest rates.

Finally, because stock prices decline on ex-dividend days, and put owners profit from such stock declines, the greater the dividend amounts on ex-days prior to expiration, the higher are put values.

In the partial derivative notation, for European put options we have:

$$\frac{\partial P}{\partial K} > 0, \quad \frac{\partial P}{\partial S} < 0, \quad \frac{\partial P}{\partial T} \gtrless 0, \quad \frac{\partial P}{\partial \sigma} > 0, \quad \frac{\partial P}{\partial r} < 0, \quad \text{and} \quad \frac{\partial P}{\partial \text{div}} > 0$$

The partial derivative $\partial P/\partial S$ is a put's delta, and $\partial P/\partial T$ is called its theta. We cannot predict how European put values will change as time passes, all else equal. Therefore, the sign of theta for European puts is indeterminate $((\partial P/\partial T)(>/<)0)$. However, American put values cannot rise in value as time passes. Thus, for American puts, $\partial P/\partial T \geq 0$.

14.8 Stock Options Markets

Listed stock options on stocks may trade on one or more of five exchanges: the Chicago Board Options Exchange (CBOE), the American Stock Exchange (AMEX), the Philadelphia Stock Exchange, the Pacific Stock Exchange, and the New York Stock Exchange. To be approved for option trading, a stock must meet specific requirements. As of 1997, the firm-specific factors that determined eligibility for trading as an underlying security to an option included:

- At least 7 million shares in the public's hands.
- At least 2000 stock owners.
- At least 2.4 million shares traded in the last 12 months.
- A closing stock price of at least $7.50/share on a majority of trading days during the last three months.
- The company must be in compliance with the rules for making timely reports as required by the Securities and Exchange Act of 1934.
- The company must not have defaulted on any interest payments, sinking fund payments, preferred stock dividends, or lease payments during the past 12 months.

An exchange can decide to discontinue trading in an option for failing to meet a set of requirements similar to the foregoing, or for any other reason, in which case trading will continue in existing options, but no new expiration dates will be introduced. If a company's stock ceases to trade—because of a merger, for example—trading in its options will cease.

The addresses of the major U.S. options exchanges are given in Table 14.4. The exchanges usually offer (free or for a small charge) books, pamphlets, courses, and videos on options and options trading. If you have never traded options before and are thinking about starting, you should contact the exchanges for the information they offer.

14.8.1 Strike Prices for Listed Stock Options

Each exchange selects option strike prices on a particular underlying stock. Typically, strikes just above and below the current market price of the stock are opened for trading. If the stock price moves above the highest strike price, the exchange will open a new series of options for all expiration months with a strike just above the previously existing highest strike. The same holds for movements below the lowest strike. For example, suppose a stock is selling for $42/share. The exchange will open for trading options with 40 and 45 strikes. If the stock were to decline below $40/share, options with a 35 strike would likely open for trading. Most strike prices are $5 apart, though low-priced stocks that sell for less than $25/share and stocks with little price volatility may have strikes just $2.50 apart, and high-priced stocks that sell for more than $200/share will have strikes $10 apart.

TABLE 14.4 Major U.S. Options Exchanges

1. Chicago Board Options Exchange
   400 South LaSalle Street
   Chicago, IL 60605
   800-OPTIONS
   www.cboe.com
2. New York Stock Exchange
   Options and Index Products
   20 Broad Street
   New York, NY 10005
   800-692-6973
   www.nyse.com
3. American Stock Exchange
   86 Trinity Place
   New York, NY 10006
   800-THE-AMEX
   www.amex.com
4. Pacific Stock Exchange
   301 Pine Street
   San Francisco, CA 94104
   415-393-4000
   800-TALK-PSE (recorded messages)
   www.pacificex.com
5. Philadelphia Stock Exchange
   1900 Market Street
   Philadelphia, PA 19103
   800-THE-PHLX
   www.phlx.com

14.8.2 Expiration Dates for Listed Stock Options

Each listed stock option falls into one of three cycles that determines what expiration months are available for trading. Basically, expiration dates exist for the most nearby two months, plus two more distant months, depending on which of the three "cycles" (January, February, or March) a stock falls into. Table 14.5 illustrates how the cycles work. For example, consider the February cycle. From the Monday after the third Friday in December until the third Friday in January,

TABLE 14.5 The Expiration Months Are Available for Trading. Each Traded Option Falls into One of These Three Cycles

I. January Cycle

From the Monday After the Third Friday in	Until the Third Friday in	Available Months			
January	February	Feb	***Mar***	Apr	Jul
February	March	Mar	Apr	Jul	Oct
March	April	Apr	***May***	Jul	Oct
April	May	May	***Jun***	Jul	Oct
May	June	Jun	Jul	Oct	***Jan***
June	July	Jul	***Aug***	Oct	Jan
July	August	Aug	***Sep***	Oct	Jan
August	September	Sep	Oct	Jan	***Apr***
September	October	Oct	***Nov***	Jan	Apr
October	November	Nov	***Dec***	Jan	Apr
November	December	Dec	Jan	Apr	***Jul***
December	January	Jan	***Feb***	Apr	Jul

II. February Cycle

From the Monday After the Third Friday in	Until the Third Friday in	Available Months			
January	February	Feb	***Mar***	May	Aug
February	March	Mar	***Apr***	May	Aug
March	April	Apr	May	Aug	***Nov***
April	May	May	***Jun***	Aug	Nov
May	June	Jun	***Jul***	Aug	Nov
June	July	Jul	Aug	Nov	***Feb***
July	August	Aug	***Sep***	Nov	Feb
August	September	Sep	***Oct***	Nov	Feb
September	October	Oct	Nov	Feb	***May***
October	November	Nov	***Dec***	Feb	May
November	December	Dec	***Jan***	Feb	May
December	January	Jan	Feb	May	***Aug***

TABLE 14.5 Continued

III. March Cycle

From the Monday After the Third Friday in	Until the Third Friday in	Available Months			
January	February	Feb	Mar	Jun	***Sep***
February	March	Mar	***Apr***	Jun	Sep
March	April	Apr	***May***	Jun	Sep
April	May	May	Jun	Sep	***Dec***
May	June	Jun	***Jul***	Sep	Dec
June	July	Jul	***Aug***	Sep	Dec
July	August	Aug	Sep	Dec	***Mar***
August	September	Sep	***Oct***	Dec	Mar
September	October	Oct	***Nov***	Dec	Mar
October	November	Nov	Dec	Mar	***Jun***
November	December	Dec	***Jan***	Mar	Jun
December	January	Jan	***Feb***	Mar	Jun

a stock in the February cycle had options expiring in January, February, May, and August. The August options were introduced on the Monday after the third Friday in December, which is why ***Aug*** is in bold italic type. On the third Friday of January, the options expiring in January cease trading. On the Monday following the third Friday of January, options that expire in March are introduced, so from that date until the third Friday in February, options expiring in February, March, May, and August trade.

The system provides that each stock has options trading that expire in the two nearby months, and also in two more distant months. The maximum expiration date possible is always eight months away.[17] Generally, the greatest liquidity is in the two nearby months.

14.8.3 Market Makers

If you wished to buy or sell an option in a world without market makers, you would have to wait for another party to contact you about taking the opposite position. This could take a long time, and might involve a lot of effort on your part.

Each exchange, however, has one or more market makers who must always stand ready to trade by quoting a bid price and an ask price. Whenever you wish to sell, you can be sure there will be a price at which a market maker is willing to buy the option (the bid price). Whenever you wish to buy options, the market maker will quote an ask price (i.e., the price at which he will sell them to you). The Chicago Board Options Exchange and the Pacific Stock Exchange have competing market makers. The American, Philadelphia, and New York Stock Exchanges use a specialist system of market making. The specialist is given the franchise of market making at these exchanges, though other competition exists for specialists.

Market makers quote bid and ask prices. The **bid** is the price at which the market maker is willing to buy the option. The **ask** is the price at which he will sell the option. The ask price always exceeds the bid price. The minimum spread between the bid and asked quotes is $0.05 for options selling for less than $3 and $0.10 for options trading for more than $3. Actively traded options will have spreads at these minima. If the option is thinly traded, i.e., low volume, the bid–ask spread will be wider. Market makers are expected to maintain fair and orderly markets. To do so, they are expected to keep bid–ask spreads narrow. The individual exchanges specify maximum spreads for options. For example, as of December 2001, the maximum quote spreads for CBOE options were: $0.25 if the option's bid quote was priced at $1.99 or less, $0.40 if the option was priced between $2.00 and $4.99, $0.50 if the option was priced between $5.00 and $9.99, $0.80 for options selling between $10.00 and $19.99, and $1 ($100 for an option on 100 shares of stock) if the last trade of the option equaled or exceeded $20. These guidelines are not binding in any way, though other exchange members grade market makers on their abilities and ethics. Those who perform poorly can be penalized. The guidelines are not applicable to the series of options with the longest time to expiration, and they do not apply if the price of the underlying asset is rapidly changing (this is termed a "fast market") or if market conditions are "unusual."

Market orders are orders placed by investors who wish to trade immediately. Thus, if an investor places a market order to buy an option, he will usually pay the ask price. An investor placing a market order to sell an option will usually receive the bid price.

The bid–ask spread is the cost of immediacy for investors who want to trade quickly. The market maker, who "makes a market" by always being available to trade, generally earns profits by buying at the bid price and selling at the ask price. Investors themselves compete against market makers when they place **limit orders**, which are orders to buy or sell at a specified price. If the existing spread is 3 bid to 3.40 asked, an investor can place a limit order to sell between the spread, say at a price of 3.30. If no trader on the floor wishes to buy the option at that price at that time, the order is entered into a book of unfilled limit orders, and the prevailing bid–ask spread becomes 3 to 3.30 (the investor's order to sell). The investor's order has supplied competition, and the spread has been narrowed.

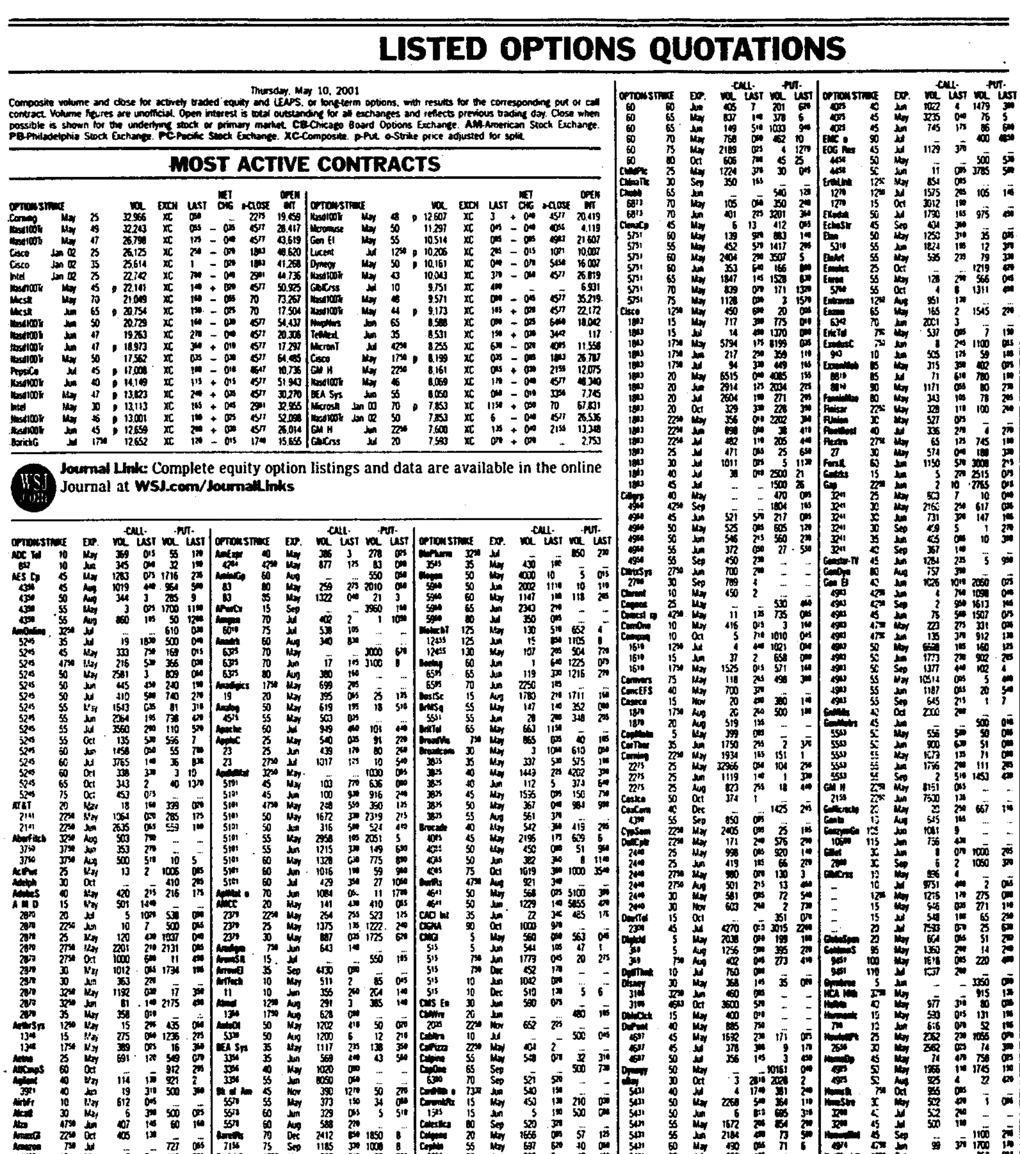
14.8.4 The Role of the Options Clearing Corporation

Options are created when a buyer and a seller agree on a price. Once a trade has been made, however, the Options Clearing Corporation (OCC) steps in and becomes a party to both sides of the trade. In other words, the option buyer purchases the option from the OCC, and the seller sells the option to the OCC. In this way, each investor is free from the worry that the other party will default on the contractual obligations. Each option investor simply looks to the OCC. The OCC itself is an agency consisting of brokerage firms called "clearing members." To guarantee the performance of all trades, the OCC has capital contributed by clearing members. If existing capital were to prove insufficient, the OCC could draw additional funds from its members, thus ensuring the integrity of the options markets.

When an option holder exercises an option, the broker contacts the OCC. Before the opening of trading on the next business day, the OCC will randomly assign the exercise to a clearing member that has a customer who is short that option. Then, the clearing firm either randomly assigns the exercise to one of its customers or assigns the option on a "first-in, first-out" basis.[18] All brokerage firms are required to inform customers which method is used. An investor should realize that an option assignment notice might arrive several days after assignment. Thus, option traders with short option positions that might be assigned should stay in contact with their brokers.

14.9 Reading Options Prices in the Financial Press

Figure 14.5 presents stock options' price data from the *Wall Street Journal* of May 10, 2001. Note that these are options on 100 shares of individual stocks. These options expire on the Saturday following the third Friday of the expiration month.



LISTED OPTIONS QUOTATIONS

Thursday, May 10, 2001

Composite volume and close for actively traded equity and LEAPS, or long-term options, with results for the corresponding put or call contract. Volume figures are unofficial. Open interest is total outstanding for all exchanges and reflects previous trading day. Close when possible is shown for the underlying stock or primary market. CB-Chicago Board Options Exchange. AM-American Stock Exchange. PB-Philadelphia Stock Exchange. PC-Pacific Stock Exchange. XC-Composite. p-Put. o-Strike price adjusted for split.

MOST ACTIVE CONTRACTS

Journal Link: Complete equity option listings and data are available in the online Journal at **WSJ.com/JournalLinks**

Figure 14.5 Some listed option quotations as presented in *The Wall Street Journal*. *Source:* (Reprinted with permission from *The Wall Street Journal*. © May 10, 2001.)

The first column gives the name of the company and its closing stock price. If options data for only one strike price are presented, no closing stock price is listed. For example, locate the entries for ActPwr and Apache. Note that the stock prices on this option page might not equal the prices given on the pages with the closing stock prices. This is because closing stock prices are "composite closing prices." After trading in equities in New York ceases, there may be additional trades on other exchanges such as the Pacific Stock Exchange, trades handled by "over-the-counter" market makers such as large brokerage houses, or on one of several new aftermarket electronic markets. The stock prices given on the options page are the closing prices on each stock's primary exchange (usually the NYSE).

The next column presents the strike prices. For Cisco Systems (Cisco), data on options with nine different strikes are presented: 12.50, 15, 17.50, 20, 22.50, 25, 30, 40, and 45. Following the strike column is a column containing the expiration month. Then, there are two columns with the volume and last trade price for calls, and two columns with volume and last trade price for puts. For example, July 20 calls on Cisco last traded at 1.90 (that is $190 on a call option covering 100 shares of Cisco), and 2604 of these calls traded on May 10, 2001. July 20 puts on Cisco last traded at 2.95, and 271 of these puts traded on May 10.

The *Wall Street Journal* provides data only for the most actively traded options. Data on all option prices can be obtained from a stockbroker, or from Internet sites such as www.cboe.com.

The exchanges introduce only "around the money" options when a new expiration month begins. Thus, at some time in the recent past, Cisco's common stock price was as low as about 14 (so that the 12.50 strikes were introduced) and as high as 41 (so that the 45 strikes commenced trading). The constitution of each exchange provides the guidelines for the procedure of introducing new option strike prices and expiration dates. The constitutions are available from the Commerce Clearing House in Chicago, or by visiting the websites of the options exchanges (e.g., www.cboe.com).

Two comments must be made about the "reliability" of the prices in Figure 14.5. The first involves the problem of nonsynchronous trading. The option prices shown are those of the last trade. However, we do not know the time of that trade. It may have been at any time between 8:30 A.M. and 3:02 P.M. (Central Standard Time), which are the times during which CBOE options on individual equities trade (index options on the CBOE trade until 3:15 P.M. Central Standard Time). Note that, from Figure 14.5, we do not know the price of the underlying stock at the time of the last option trade. We only know the closing stock price. Since most stocks are more actively traded than their options, it is likely that most option prices reflect information that existed earlier in the day than when the stock last traded.

The second problem concerns bid–ask spreads. We do not know whether any given price in Figure 14.5 was that of an investor's market buy order at the asked price or an investor's market sell order at the bid price, or a price within the bid–ask spread. Of course, owing to nonsynchronous trading, the actual bid–ask spread at the close of trading might be considerably different from the bid–ask spread that existed at the time of the last trade.

14.10 Transaction Costs

There are two primary types of transaction cost. One is termed **market impact**. Market impact consists of the bid–ask spread and price pressure. Placing market orders to buy a security will usually be filled at the ask price, while market orders to sell will be done at the bid price. The spread represents the cost of immediacy (the price an investor pays for an immediate trade). An investor can *try* to overcome this cost by placing a limit order, but at the risk of not having the order filled. The bid and asked prices quoted by market makers are good only for specified numbers of options. Price pressure occurs when an investor tries to trade a large number of options. For large trades, higher purchase prices and/or lower sale prices can result.

The other important transaction cost is the broker's commission. There is a range of commissions for options, much of which is determined by whether an investor deals with a discount broker or a full-service broker. In addition, trades entered electronically over the Internet usually have lower commissions than trades that involve a broker or phone representative. The options commission schedules of two discount brokers are shown in Tables 14.6 and 14.7. Even though both call themselves discounters, you can see that there is a substantial difference in their rates for inactive traders. The minimum commission on any options trade done through broker A and broker B is $25 and $36, respectively. But broker B's commissions for active traders are very competitive with those of A.

Full-service brokers such as Merrill Lynch and Morgan Stanley usually charge more than discount brokers. However, the investor receives personalized service for the higher commissions. An account executive is assigned to each investor and will discuss specific stock or option recommendations (which, generally speaking, are probably no better than any stocks or options you could randomly pick). In addition, a full-service broker will be able to provide more information about the security, planned and potential trades, and general market conditions. A good account executive will be able to minimize your trading costs after assessing your needs by determining your

TABLE 14.6 Price Schedule for Discount Broker A

Market order up to and including 30 contracts	$15 + $1.50 per contract
Market order over 30 contracts	$15 + $1.75 per contract
Limit orders	$15 + $1.75 per contract
All broker-assisted option orders	$25 + $1.75 per contract
Minimum commission: $25.00	

TABLE 14.7 Price Schedule for Discount Broker B

Transaction Amount	Commission Rate
$0–$2500	$28.75 + 1.60% of principal
$2501–$10,000	$48.75 + .80% of principal
> $10,000	$98.75 + .30% of principal

Minimum: $34.25 plus $1.75 per contract (a minimum of $36, but not to exceed maximum charge).

Maximum: $36 per contract for the first two contracts, plus $4 per contract thereafter; or half the principal amount (whichever is less).

There is a 25% discount to the amounts above for electronically placed orders.

Accounts that trade stocks, bonds, or options 36 or more times in a rolling 12-month period face a minimum commission of $27, or the following:

Trades electronically placed	$20 + $1.75 per contract
Trades placed via a phone representative	$35 + $1.75 per contract

level of risk aversion and your need for current income versus your need to invest for retirement. A good broker will call you when important news has been released. He or she may warn you about the likelihood of being assigned an option in the near future, or about the advisability of exercising or selling a given option. Full-service brokers may also offer a wider range of products than discounters. Unfortunately, many full-service account executives also make their living from your trading costs, which can cause a conflict of interest. In contrast, discounters such as broker A are little more than order intermediaries. Deep-discount brokers will give you some price information, take orders, and pass them on to their floor representatives at the appropriate exchanges.

14.11 MARGIN

The practice of borrowing money from a broker to buy securities is known as "buying on margin." Investors can buy most securities on margin. If the Federal Reserve Board sets the initial margin requirement at $x\%$, then an investor must put up at least $x\%$ of the purchase price in cash and can borrow at most $(1-x)\%$ from the broker. Currently, the initial margin on stock purchases is 50%.

An investor must have at least $2000 to open a margin account. Thus, with **initial margin** at 50%, an investor can buy as much as $4000 worth of stock with that money (ignoring brokerage fees).

Exchanges and brokers also set **maintenance margin** requirements, which become effective after the securities have been purchased. The maintenance margin requirement fixes the point at which the investor receives a margin call. If the value of the collateral for the loan has declined, a **margin call** commands the investor to deposit additional cash or securities into the account. That is, suppose the maintenance margin requirement is $z\%$. Then, if an investor's equity position falls to $z\%$ of the current market value of the account, the investor must place additional cash and/or securities into the account to increase the account balance to the initial margin level.

Buying on margin allows an investor to create leverage. That is, if an investment does well, higher rates of return will be earned. In Example14.3, if the stock price increases to $60/share,

EXAMPLE 14.3 Suppose that the initial margin requirement is 60%, and the maintenance margin requirement is 25%. An investor can buy $10,000 worth of stock with $6000 of his own money and borrow $4000 from his broker. Assume the purchase is for 200 shares of a stock selling for $50/share, and ignore all commissions. Suppose the stock price declines to $26.625/share, so that the total value of the stock in the account is $5325. The investor's equity in the account is reduced to $1325. That is, if the investor sold the 200 shares, he would receive $1325 after repaying his loan. Thus, the investor's margin now is only ($1325/$5325 =) 24.88%. The investor would have to deposit additional cash and/or securities to increase the account balance. If the investor does not act quickly, the broker will sell shares to ensure repayment of the margin loan. Note that for simplicity in this example, the interest on the loan has been ignored. The effect of accrued interest is that the investor would have received the margin call at a stock price higher than $26.625.

which is a 20% increase in price, the investor earns a 33.33% rate of return on the \$6000 investment (again ignoring commissions and the repayment of interest). However, leverage is a double-edged sword. If the margin requirement is 60%, then any x% decline in the stock's price will result in a loss of x/0.6%. If the stock in Example14.3 declined 20% to \$40/share, an investor who purchased the stock on margin will have lost 33.33% on his \$6000 investment.

Option purchases are already levered positions. As such, the Federal Reserve's Regulation T states that all options purchases must be for cash only. Investors cannot borrow from brokers to purchase options. At times, however, investors must post margin to maintain an option position—for example, when an investor sells an option but does not have a position in the underlying shares. This practice is known as "selling naked options."

The sale of naked options is risky. Losses are theoretically unlimited on written naked calls, and a writer of naked puts is obligated to pay the strike price (\$$K$) for the purchase of the stock, should a put owner exercise it. Brokers, the exchanges, and the Fed want assurance that writers of naked option will fulfill their obligations. Consequently, there are rather substantial margin requirements on written naked positions. Indeed, some brokers will not handle requests to write naked options at all, and others demand that clients have substantial equity in their accounts before allowing them to trade in this way.

At the CBOE, the initial margin required to write a naked call option on common stock was, as of 2001 (it is always subject to change):

$$\max\{C+0.1S,\ C+0.2S-(\max[0,K-S])\}$$

where C is the call premium, S is the price of 100 shares of the underlying stock, K is the strike price, and $\max[0, K-S]$ represents the amount, if any, by which the call is out of the money.

This formula says that the required margin is either (a) the market value of the call plus 10% of the market value of the stock or (b) the market value of the call plus 20% of the market value of the stock minus any out-of-the-money amount. The required margin is the greater of (a) and (b).

EXAMPLE 14.4 Suppose an investor writes one naked June 17.50 Cisco put for \$1.10, when the stock is selling for \$18.83/share (see price data in Figure 14.5). The margin required is:

$$\max\{110+0.1(1883),\ 110+0.2(1883)-133\}$$

$$\max\{298.3, 353.6\} = \$353.60$$

While this might seem small relative to the funds involved in a trade of 100 shares of Cisco stock (\$1883), keep in mind that this put has no intrinsic value. If price of Cisco remains above the put's strike price, the option will expire worthless. Even if a put is in the money, and the writer of a naked put is assigned the option, the writer can always then sell the assigned stock immediately,[19] and the realized loss will be only the put's intrinsic value. From the exchange's viewpoint, the margin amount protects the exchange from the risk that the put writer will default when assigned the stock. The amount at risk is only the put's intrinsic value.

Similarly, the CBOE initial margin requirement for writing naked puts on common stock is:

$$\max\{P+0.1S,\ P+0.2S-(\max[0,S-K])\}$$

where P is the put premium, S is the price of 100 shares of the underlying stock, K is the strike price, and $\max[0, S-K]$ represents the amount, if any, by which the put is out of the money.

Writers of naked options can use cash to satisfy the margin requirement, or they can use securities. However, the required market value of the securities differs, depending on the nature of the collateral. That is, the market value of Treasury securities or common stock used to meet initial margin requirements will almost certainly be greater than the $353.60 in cash that is required to write the Cisco put of Example 14.4.[20]

In one commonly used strategy, the **covered call**, an investor owns the stock, and sells a call. In this situation, margin is required on the stock position, but there is no margin required for the short call position. Indeed, the call premium received from the sale of the call can be used to reduce the margin required on the stock.

If the call is in the money, then the investor can borrow (i.e., by buying on margin) only 50% of the strike price, plus the premium received from the written call. In example 14.5, if $S=50$, $K=45$ and $C=6\ 1/2$, the investor can borrow as much as $(0.5\times\$4500+\$650=)$ \$2900. This maximum borrowing amount is fixed as long as the call remains in the money.

Margin requirements can become quite complex when more complex strategies, such as those discussed in the next chapter, are used. Investors should consult their brokers to learn the rules for margin requirements if any of these strategies are used.

14.12 TAXES

What follows describes some of the basic tax rules for option positions. Most option profits are taxed as ordinary income. Therefore, depending on the option trader's income, the marginal tax rate on option profits is 15, 28, 31, 36, or 39.8%. Tax laws frequently change, however, and they can be quite complex for some types of transaction. Thus, an option trader should consult a recent tax guide, a tax accountant, or a tax attorney before doing his taxes.

Rule 1. Open and then close (at a subsequent date) a position of options on common stock. When the position is closed, there is a capital gain or a capital loss. The option trader must pay taxes on any gain (less commissions) at his marginal tax rate in the year the position is closed.

Total capital losses of less than $3000 in one year will lower your taxable income for that year. In Example14.6, if you sold the put for $300 in November 1999, and offset that trade by buying the put for $425 in January 2000, there would be a net capital loss after commissions of

EXAMPLE 14.5 An investor buys 100 shares of a $50 stock on margin and writes an at-the-money or out-of-the money call for $212.50. Ignoring commissions, and assuming that the initial margin requirement is 50%, the investor is required to deposit $(0.5\times\$5000-\$212.50=)$ \$2287.50 to meet the initial margin requirement on the position.

(\$125 + \$42 =) \$167. Taxable income would be reduced by this amount. If your marginal tax rate is 28%, then your 2000 tax obligation would be reduced by \$46.76 (0.28 × \$167).

Total capital losses exceeding \$3000 in one year can in some cases be carried back for three years, if they can be applied to gains made on the same types of security during those years. Otherwise, capital losses above \$3000 must be carried forward to future years.

Rule 2. Open and close a position on index options or futures options. If the trades both occur during the same year, the loss or gain is taxed as just described. If the position is carried from one year to the next, the investor must "mark to market" on the last trading day of the year. Index options and futures options are referred to as "Section 1256 contracts," which means that they must be "marked to market" at the end of the year for tax purposes. Example 14.7 illustrates this rule.

Rule 3. Open a long position and then exercise your option. Alternatively, you may open a short position and later be assigned the option. The basis of the position taken in the underlying security is adjusted by the original option premium. Taxes are deferred until disposal of the underlying asset. Example 14.8 illustrates this rule.

EXAMPLE 14.6 You sell a naked put for \$300 in November 2001 and offset the trade by buying the same put for \$75 in January 2002. There is a capital gain of \$225. Total commission on the two trades is \$42, so the net capital gain is \$183. If your marginal tax rate is 28%, then \$51.24 is the tax due on the gain. This tax is payable on your 2002 tax return.

EXAMPLE 14.7 You buy an S&P 500 Index option call for \$6.10 in December 2001. At the close of the last trading day of 2001, the call closes at 5. You realize a loss of \$110 for tax purposes in 2001 [(5 − 6.10)100]. Your basis for the call purchase now becomes 5. If you subsequently sell the call for \$6 in January 2002, you have a taxable profit of \$100 in 2002.

EXAMPLE 14.8 You buy a call on 100 shares of stock for \$300. You subsequently exercise the call and buy the stock for the aggregate strike price of \$5000. For tax purposes, your purchase price of the stock is \$5300 (plus all commissions). Tax effects are realized only when the stock is later sold. This can make for an effective tax strategy. Suppose you buy a December call in August. The stock price rises sharply between August and early December. If you sell the call in the first three weeks in December, you will realize a large taxable gain in the current year. Instead, you may wish to exercise the call and hold the acquired stock until January. By selling the stock in January, you will have postponed the taxes on the gain for a year. Of course, you will also bear price risk, since the price of the stock can change during December.

After reading Example 14.8, you might be asking, "Why not buy a put or sell a deep-in-the-money call (after exercising the original call) to lock in the gain, and then wait until the next year to offset all the positions?" This would remove much of the price risk that exists between the exercise date and the January selling date. However, this may or may not be allowable. Rather than discuss all the possible ways one can postpone paying taxes on gains and hasten the tax inflows because of losses, you are urged to investigate before you act.[21]

14.13 INDEX OPTIONS

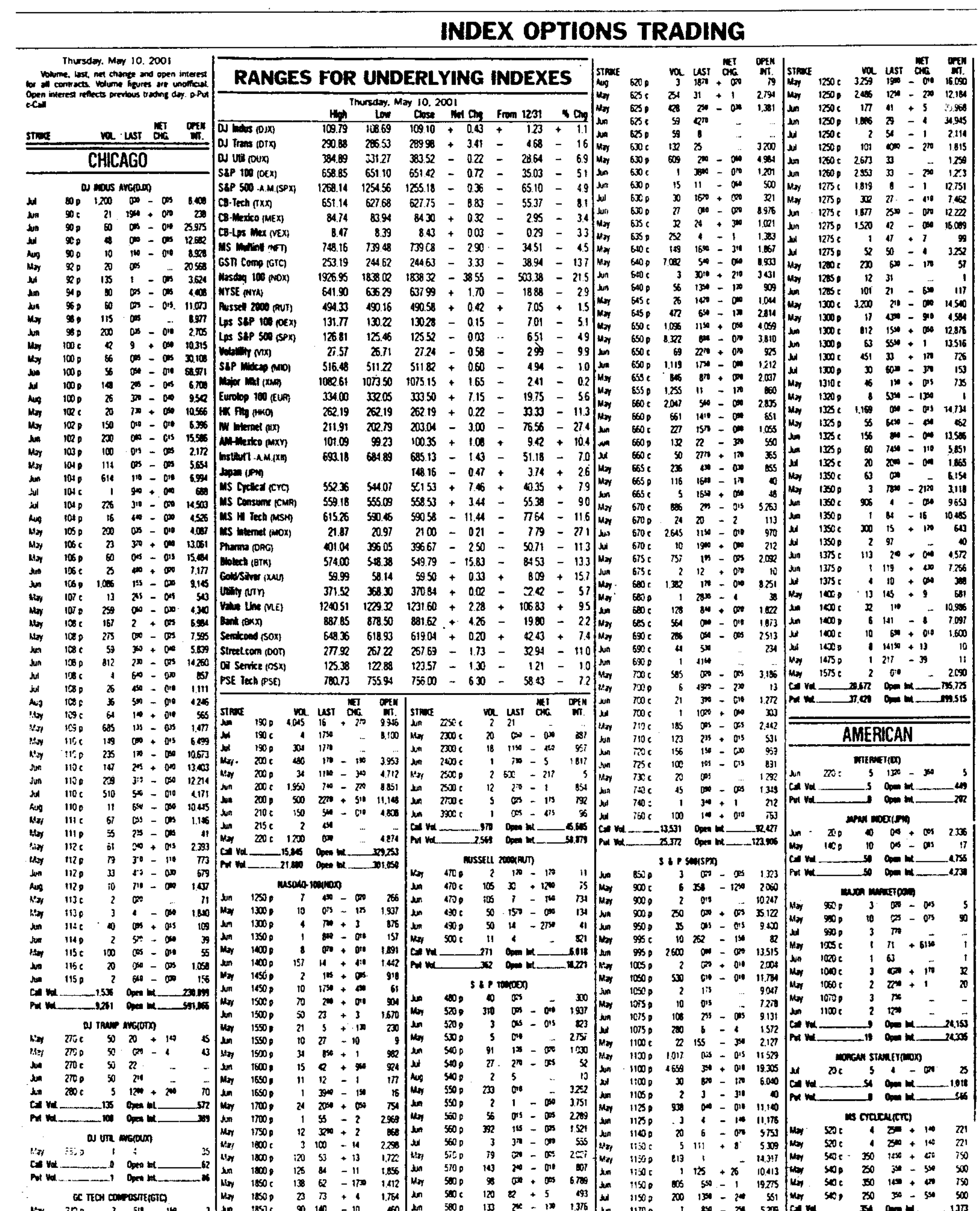
Index options began trading in the United States on March 11, 1983. The first index option was the CBOE 100, which was later renamed the S&P 100. It is now also frequently called the OEX, which is its ticker symbol. By the end of 1983, options on different index traded not only on the CBOE but on the American, Philadelphia, and New York Stock Exchanges. Figure 14.6 shows how price data on index options are presented in the *Wall Street Journal*. Other index options currently exist, however there is insufficient volume for the financial press to provide price data.

The index option with the largest open interest is on the S&P 500 Index. Its ticker symbol, and common name, is SPX. Listed options on portfolios of stocks concentrated in certain sectors such as banking stocks, drug stocks, and Internet stocks, have been introduced, and trading information on some of these are also shown in Figure 14.6. The NYSE once tried to develop interest in an option on a portfolio of the NYSE stocks with the highest "beta"[22]. Every quarter, the NYSE ranked all listed stocks by their relative volatility (beta) and grouped the highest of them into a portfolio called the beta index. Thus, investors who wished to speculate by trading options on the riskiest stocks on the NYSE could do so. The option could also be expected to serve the hedging needs of certain investors holding portfolios of risky stocks. However, the option on the beta index was not a success and trading ceased at the end of 1987. In 1991 the CBOE introduced OEX LEAPS (long-term equity anticipation securities), and SPX LEAPS. These instruments have times to expiration as long as three years. The strike prices and index values of these index LEAPS equal one-tenth of the levels of the S&P 100 and S&P 500 indexes. For example, if the S&P 500 Index is at 1300, the S&P 500 LEAP Index equals 130.0. Strike prices would be around 125, 130, and 135.

Index options offer investors an efficient way to speculate on the future direction of the stock market. Some investors possess better timing skills than stock selection skills. They may be correct in predicting when the stock market will rise but tend to buy stocks (or options on individual stocks) that underperform the market. Index options also offer a low cost way to effectively buy the market today when a large cash inflow, not currently available, is nevertheless anticipated. Index options can be used to hedge an existing portfolio against a systematic decline in equity values. Note that there are a few differences between index options and individual equity options, and they are discussed in the sections that follow.

14.13.1 Index Options Are Cash Settled

Listed index options do not permit the delivery of the underlying asset. It would be quite difficult and costly to accumulate each of several hundred stocks in their proper weights to make or take delivery. Instead, when index options are exercised, they are settled by a cash payment. The cash amount exchanged when a call is exercised equals the index value minus the strike price, times 100.



INDEX OPTIONS TRADING

Thursday, May 10, 2001

Volume, last, net change and open interest for all contracts. Volume figures are unofficial. Open interest reflects previous trading day. p-Put c-Call

RANGES FOR UNDERLYING INDEXES

Thursday, May 10, 2001

	High	Low	Close	Net Chg	From 12/31	% Chg
DJ Indus (DJX)	109.79	108.69	109.10	+ 0.43	+ 1.23	+ 1.1
DJ Trans (DTX)	290.88	286.53	289.98	+ 3.41	– 4.68	– 1.6
DJ Util (DUX)	384.89	331.27	383.52	– 0.22	– 28.64	– 6.9
S&P 100 (OEX)	658.85	651.10	651.42	– 0.72	– 35.03	– 5.1
S&P 500 -A.M.(SPX)	1268.14	1254.56	1255.18	– 0.36	– 65.10	– 4.9
CB-Tech (TXX)	651.14	627.68	627.75	– 8.83	– 55.37	– 8.1
CB-Mexico (MEX)	84.74	83.94	84.30	+ 0.32	– 2.95	– 3.4
CB-Lps Mex (VEX)	8.47	8.39	8.43	+ 0.03	– 0.29	– 3.3
MS Multintl (NFT)	748.16	739.48	739.08	– 2.90	– 34.51	– 4.5
GSTI Comp (GTC)	253.19	244.62	244.63	– 3.33	– 38.94	– 13.7
Nasdaq 100 (NDX)	1926.95	1838.02	1838.32	– 38.55	– 503.38	– 21.5
NYSE (NYA)	641.90	636.29	637.99	+ 1.70	– 18.88	– 2.9
Russell 2000 (RUT)	494.33	490.16	490.58	+ 0.42	+ 7.05	+ 1.5
Lps S&P 100 (OEX)	131.77	130.22	130.28	– 0.15	– 7.01	– 5.1
Lps S&P 500 (SPX)	126.81	125.46	125.52	– 0.03	– 6.51	– 4.9
Volatility (VIX)	27.57	26.71	27.24	– 0.58	– 2.99	– 9.9
S&P Midcap (MID)	516.48	511.22	511.82	+ 0.60	– 4.94	– 1.0
Major Mkt (XMI)	1082.61	1073.50	1075.15	+ 1.65	– 2.41	– 0.2
Eurotop 100 (EUR)	334.00	332.05	333.50	+ 7.15	– 19.75	– 5.6
HK Fltg (HKO)	262.19	262.19	262.19	+ 0.22	– 33.33	– 11.3
IW Internet (IIX)	211.91	202.79	203.04	– 3.00	– 76.56	– 27.4
AM-Mexico (MXY)	101.09	99.23	100.35	+ 1.08	+ 9.42	+ 10.4
Institut'l -A.M.(XII)	693.18	684.89	685.13	– 1.43	– 51.18	– 7.0
Japan (JPN)			148.16	– 0.47	+ 3.74	+ 2.6
MS Cyclical (CYC)	552.36	544.07	551.53	+ 7.46	+ 40.35	+ 7.9
MS Consumr (CMR)	559.18	555.09	558.53	+ 3.44	– 55.38	– 9.0
MS Hi Tech (MSH)	615.26	590.46	590.58	– 11.44	– 77.64	– 11.6
MS Internet (MOX)	21.87	20.97	21.00	– 0.21	– 7.79	– 27.1
Pharma (DRG)	401.04	396.05	396.67	– 2.50	– 50.71	– 11.3
Biotech (BTK)	574.00	548.38	549.79	– 15.83	– 84.53	– 13.3
Gold/Silver (XAU)	59.99	58.14	59.50	+ 0.33	+ 8.09	+ 15.7
Utility (UTY)	371.52	368.30	370.84	+ 0.02	– 22.42	– 5.7
Value Line (VLE)	1240.51	1229.32	1231.60	+ 2.28	+ 106.83	+ 9.5
Bank (BKX)	887.85	878.50	881.62	+ 4.26	– 19.80	– 2.2
Semicond (SOX)	648.36	618.93	619.04	+ 0.20	+ 42.43	+ 7.4
Streetcom (DOT)	277.92	267.22	267.69	– 1.73	– 32.94	– 11.0
Oil Service (OSX)	125.38	122.88	123.57	– 1.30	– 1.21	– 1.0
PSE Tech (PSE)	780.73	755.94	756.00	– 6.30	– 58.43	– 7.2

Figure 14.6 Some index options trading prices from *The Wall Street Journal*. *Source:* (Reprinted with permission of *The Wall Street Journal*. © May 10, 2001.)

If an index option is exercised before its expiration day, the index value used for cash settlement is the closing value on the day it is exercised. If the index option finishes in the money on its expiration day, the index value for final cash settlement is determined and used to define the cash flow from the option writer to the owner.

For example, if an SPX call with a strike of 1350 expires, and the closing spot SPX index is at 1352.40, the owner of each call receives $240 from the writer of each call.

Note that over time exchanges have changed the method of computing the final settlement value. For example, the CBOE has changed the final settlement for the SPX from closing prices to opening prices. Exchanges have also settled options on days other than the third Friday of the month and have used multipliers other than 100. Thus, as with all derivatives, investors should always be aware of the current contractual terms.

Investors who exercise their American-style index options early should do so as late in the day as possible. The cash settlement is based on the *closing* index value on the early exercise day. The early exerciser faces risk that the index will change in value between the time he instructs his broker to exercise the option and the close. It is even possible that an irrational early exerciser will have to *pay* the option writer. For example, the owner of an S&P 100 put with a strike price of 735 might exercise his option at 10 A.M. when the S&P 100 Index is at 733, expecting to be paid $200. However, if the S&P 100 Index closed at 736, the put owner would actually have to pay $100 to the person who is assigned the option!

Adding to the risks of exercising listed index options is the institutional fact that many brokers have specific cutoff times for customers who wish to exercise their options. For example, a broker might demand that any customer who exercises an option do so before noon.

14.13.2 Timing Risks

The writers of listed options will typically not learn that they have been assigned options until the day *after* the actual assignment day that determines the required cash flow. The writers' stockbrokers will be informed before trading commences on the day after the assignment. A competent stockbroker will then immediately inform his customers. However, not all brokers are so conscientious. Also, a writer who cannot receive the phone call will not learn of the assignment until after trading has commenced on the day after assignment. This creates some unique risks for index option writers.

For example, consider an investor who owns a highly diversified portfolio of stocks similar to those in the S&P 100 Index and writes an OEX call. The purpose of this strategy is to try to write a covered call. Suppose that the investor is assigned the call and is therefore liable for a cash payment to the exerciser. At best, the investor will not be informed of the assignment until just before the market opens on the day after his assignment cash flow liability has been established. But by then, the hedge position (long stock and short call) is no longer in effect because the investor is only long stock. Should the stocks open lower, the investor will have to sell the stocks at prices lower than the cash settlement reflected.

The writer of a covered call on a specific stock faces no such risk. Should he be assigned, he will merely deliver the stock he owns. Here, the option writer does not care if the stock's value has changed.

Similar timing risks face investors who use some of the strategies described in the next chapter. In a spread, for example, the investor buys one option and sells another option with a different strike price. With options on individual stocks, if the written option is assigned, an investor can merely exercise the long position to satisfy the settlement obligation. With index options, if the index value moves adversely between the time the option is exercised and the time the investor learns of the assignment, losses might accumulate.

14.14 FOREIGN EXCHANGE OPTIONS

Foreign exchange (FX) options have traded on the Philadelphia Stock Exchange (PHLX) since 1982. Exchanges outside the United States also list FX options. Calls on foreign exchange give the owner the right to buy the stated amount of foreign exchange at the strike price. The strike price is itself an exchange rate. Foreign exchange puts give the owner the right to sell FX at the strike price. The exchange traded currency options market is quite illiquid; the OTC currency options market is geared toward institutions and is more active. To become more competitive, the PHLX has introduced customized currency options; you can read more about these innovative contracts at the exchange's website (www.phlx.com/products/currency.html). The prices (premiums) of the contracts that trade on the PHLX are presented in the *Wall Street Journal's* "Currency Trading" section. An example of the price data is shown in Figure 14.7.

Six types of spot FX trade on the PHLX: Australian dollars, British pounds, Canadian dollars, Japanese yen, Swiss francs, and euros. Each of these are options on the $/FX exchange rate. Customized options on "cross-rate" FX options also trade on the PHLX. Finally, note that both European and American FX options trade on the PHLX. European FX options, which have been traded on the PHLX only since August 1987, are specifically noted in the *Wall Street Journal* reprint in Figure 14.7. If there is no designation, the price data are for American options. Two expiration dates are offered: midmonth and end of month. Besides its customized options, the PHLX has also been quite innovative by offering long-term options with expirations two years in the future and options that expire on a weekly basis (which are useful for hedging weekend currency risk exposure). Additional information on PHLX FX options is available at its website (www.phlx.com).

Figure 14.7 shows only data for options with sufficient volume to be reported in the *Wall Street Journal*. Observe the euro options in Figure 14.7. The underlying asset is €62,500. On May 15, 2001, the spot price of a euro was $0.8815/€. Two strikes are reported for European-style calls expiring in May: 88 and 90. The 88 strike is for 88 cents/euro ($0.88/€). The May 88 call closed at 0.47 cent per euro. Multiply that by the €62,500 underlying the FX call, and the price per option is $293.75. At expiration, the payoff of the May 88 call is:

$$C_T = 0 \qquad \text{if } S_T \le \$0.88/€$$
$$C_T = €62{,}500(S_T - \$0.88/€) \qquad \text{if } S_T > \$0.88/€$$

where S_T is the expiration day exchange rate, expressed as $/€. The May 88 call is in the money and has an intrinsic value of 0.15, or ($0.0015/€)×€62,500=$93.75. The remainder of the call premium is time value.

Besides these listed FX options, there is a huge OTC market for customized currency options. The decision maker should consider such factors as liquidity, flexibility in designing the contract to most precisely meet needs, transactions costs, relative pricing, and advice and/or any other business relationship that may be gained by dealing with the OTC dealer. In addition, the decision maker must weigh the risk that the Options Clearing Corporation will default versus the risk that the OTC option dealer will default. Both are probably small, but ratings agencies such as Standard & Poors can provide guidance.